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SUMMARY

Surface temperatures were calculated for bodies in steady supersonic flight at Mach numbers from 2 to 10 and for altitudes from 50,000 to 200,000 feet and emissivities from 0 to 1. The importance of the effects of radiation and convection was determined. It was found, under the assumption of an isothermal atmosphere, that the gain of heat from the air by convection decreases at constant Mach number as the altitude is increased. Equilibrium between convection and radiation is established at temperatures that consequently decrease as altitude is increased. In general, therefore, at sufficiently high altitudes the surface temperature is considerably less than the stagnation temperature. At a Mach number of 8, for example, the stagnation temperature is 4800° F absolute and the equilibrium surface temperature for an emissivity of 0.5 is 3800° F absolute at 50,000 feet and decreases to 1350° F absolute at 200,000 feet.

INTRODUCTION

As actual and proposed speeds of flight increase, the problem of aerodynamic heating of airplanes and rockets becomes a source of great concern. The importance that the problem assumes is forcibly illustrated by the heating of meteors. While a meteor is moving through empty space, its temperature is low. Yet, within a few seconds after the meteor enters the earth's atmosphere, its surface has become incandescent. Although combustion doubtless plays a role in the final temperature reached, the surface of the meteor attains at least combustion temperature by friction and by compression of the earth's atmosphere.

The purpose of the present paper is to examine the problem of aerodynamic heating of bodies in steady supersonic flight, to state what alleviating factors exist, to determine the significance of these factors and, finally, to calculate the temperatures that can be expected on the surfaces of bodies at supersonic speeds.

The results presented herein are based on analysis and on extrapolation. It was found necessary to use subsonic heat-transfer equations for supersonic flow, small-viscosity-gradient heat-transfer equations in flow where the viscosity gradient is large, and estimated atmospheric density at altitudes at which the properties of the atmosphere are not yet accurately known. The extrapolations are necessary at the present time because of a lack of experimental and theoretical data. The quantitative accuracy of the results obtained herein depends, of course, on the correctness of the several assumptions that were necessary in making the extrapolations.

SYMBOLS

a	ratio of characteristic temperatures (θ_0/θ_N)
c_p	specific heat at constant pressure, Btu/(lb)(°F)
c_v	specific heat at constant volume, Btu/(lb)(°F)
D	effective molecular diameter $(9.1 \times 10^{-10}$ ft)
g	acceleration due to gravity (32.2 ft/sec^2)
h_x	local coefficient of heat transfer at distance x from leading edge, Btu/(sec)(sq ft)(°F)
$\bar{h}$	average coefficient of heat transfer over distance x from leading edge, Btu/(sec)(sq ft)(°F)
H	rate of heat transfer, Btu per second
J	mechanical equivalent of heat (778 ft-lb/Btu)
k	thermal conductivity, Btu/(sec)(sq ft)(°F/ft)
M	Mach number
Pr	Prandtl number $(c_p \mu g/k)$
R	gas constant (for air, 53.52 ft-lb/(lb)(°F abs.))
Re	Reynolds number $(\rho V x/\mu)$

S	surface area, square feet
T	temperature, °F absolute
T_s	temperature of surface, °F absolute
T_t	total or stagnation temperature, °F absolute
V	velocity, feet per second
x	distance from leading edge, feet
Y	altitude, feet
z	ratio of temperatures (θ_N/T)
α	shock-wave angle, degrees
β	half angle of wedge airfoil, degrees
γ	ratio of specific heats (c_p/c_v)
δ	boundary-layer thickness, feet
ϵ	emissivity, ratio of emissive power of actual surface to that of black body
θ	characteristic temperature, °F absolute
λ	mean free path, feet
μ	coefficient of viscosity, slug/(ft)(sec)
υ	number of molecules per unit volume $(7.3 \times 10^{23}$ per cubic foot at sea-level density$)$
ρ	mass density, slug per cubic foot
σ	Stefan-Boltzmann constant $\left(4.8 \times 10^{-13}\ \text{Btu/(sec)(sq ft)(°F abs.)}^4\right)$

Subscripts:

N	nitrogen
O	oxygen
1,2	regions in figure 4
t	total

ANALYSIS

In order to calculate surface temperatures, stagnation or total temperatures are first calculated. The variation of the total or stagnation temperature of air with Mach number is shown in figure 1. Figure 1 applies in the stratosphere, under the assumption that the atmospheric temperature there is 392° F absolute. The equation

$$T_t = T\left(1 + \frac{\gamma - 1}{2} M^2\right) \qquad (1)$$

with $\gamma = 1.4$ was used to obtain figure 1. The values of stagnation temperature shown in figure 1 are frequently thought of as the temperatures that will exist on the surfaces of supersonic bodies. Even at a Mach number of 4 the stagnation temperature is high and rises rapidly as Mach number is increased. The use of the values of stagnation temperature given in figure 1 as the surface temperatures of airplanes and rockets, however, is an oversimplification of the problem, as certain factors cause the surface temperatures to be less than the temperatures shown in figure 1.

Variation of Specific Heats

In the first place, the specific heats of air are functions of the temperature. A more refined value of stagnation temperature than that given by equation (1) can therefore be obtained by taking into account the variation of the specific heats with temperature. According to the classical kinetic theory of gases, the specific heats of (diatomic) air are

$$c_p = c_v + \frac{R}{J} = \frac{7}{2}\frac{R}{J} \qquad (2)$$

at temperatures near 500° F absolute. At these temperatures the three degrees of freedom due to translation of the molecules and the two degrees of freedom due to rotation have been fully brought into play. As the temperature is raised, vibration of the molecules is gradually activated and makes its contribution to the specific heats. As the temperature increases, therefore, the values of c_p and c_v increase above the values given by equation (2), and the value of γ decreases below the value of 1.4 that was used in equation (1) and in figure 1. If the temperature is raised enough, dissociation also contributes to the specific heats. The effect of dissociation, however, is not included in the present analysis, as the effect is not very important at the temperatures considered herein and a quantitative treatment of the effect would be rather complex. Electronic excitation and molecular and atomic ionization are also present if the temperature is raised enough. Of these processes the electronic excitation of oxygen occurs at the lowest temperature and has the largest effect on the specific heats. A check calculation shows, however, that its contribution to the specific heats is only one-half of 1 percent at 6000° F absolute. The effects of excitation and ionization are therefore not taken into account herein. Vibration, however, begins to occur at approximately 500° F absolute, and the effect of vibration on the specific heats and the stagnation temperature is therefore investigated.

The specific heat of air at constant pressure is given by the following equation, in which the quantities on the left-hand side refer to air, which is considered to be composed of 79 percent nitrogen by volume and 21 percent oxygen. Thus,

$$\frac{c_p}{R} = 0.79\,\frac{c_{p_N}}{R_N} + 0.21\,\frac{c_{p_O}}{R_O} \tag{3}$$

According to the classical quantum statistics (see, for example, reference 1),

$$J\,\frac{c_{p_N}}{R_N} = \frac{7}{2} + \frac{z^2 e^z}{(e^z - 1)^2} \tag{4}$$

$$J\,\frac{c_{p_O}}{R_O} = \frac{7}{2} + \frac{(az)^2 e^{az}}{(e^{az} - 1)^2} \tag{5}$$

where

$$z = \frac{\theta_N}{T}$$

and

$$az = \frac{\theta_O \theta_N}{\theta_N T}$$

The quantities θ_N and θ_O are the Planck-Einstein "characteristic temperatures." From measurements of molecular spectra they are known to have the values (see, for example, reference 2)

$$\theta_N = 6060^\circ \text{ F absolute}$$

$$\theta_O = 4050^\circ \text{ F absolute}$$

By use of equations (4) and (5) equation (3) can then be rewritten as

$$J \frac{c_p}{R} = \frac{7}{2} + 0.79 \frac{z^2 e^z}{(e^z - 1)^2} + 0.21 \frac{(az)^2 e^{az}}{(e^{az} - 1)^2} \tag{6}$$

The expression for stagnation temperature that takes into account the variation of the specific heat of air is derived as follows: The differential equation that expresses conservation of energy in an adiabatic flow process is

$$c_p gJ \, dT + V \, dV = 0 \tag{7}$$

If substitutions are made in equation (7) for c_p from equation (6) and for dT from the relation

$$dT = - \frac{\theta_N \, dz}{z^2}$$

then

$$- \frac{3.5 \, dz}{z^2} - \frac{0.79 e^z \, dz}{(e^z - 1)^2} - \frac{0.21 a e^{az} \, d(az)}{(e^{az} - 1)^2} + \frac{V \, dV}{gR\theta_N} = 0 \tag{8}$$

Integration of equation (8) between the limits of the free-stream condition (subscript 1) and the stagnation condition (subscript t) gives

$$\frac{3.5}{z_t} + \frac{0.79}{e^{z_t} - 1} + \frac{0.14}{e^{0.67z_t} - 1} = \frac{3.5}{z_1} + \frac{0.7M_1^2}{z_1} \qquad (9)$$

in which extremely small terms on the right-hand side have been omitted, $\gamma_1 gRT_1M_1^2$ has been substituted for V_1^2, and numerical values have been substituted for a and γ_1. Equation (9) gives the relation between stagnation temperature $T_t = \frac{\theta_N}{z_t}$ and free-stream Mach number M_1. Stagnation temperature as a function of Mach number, as given by equation (9) for a free-stream temperature of 392° F absolute, is shown in figure 2. At the lower values of Mach number the deviation of the stagnation temperature from the values shown in figure 1 is very small. At the higher Mach numbers the deviation is larger and amounts to 1300° at a Mach number of 10. The numerical computations of surface temperature for the present paper were based on the stagnation temperatures shown in figure 2.

Recovery Factor

The second factor that has a bearing on the temperatures obtained in flight is the so-called "temperature recovery factor". If a fluid is brought from a state of motion to a state of rest by an adiabatic compression, as at a stagnation point, its temperature is raised, as shown in figure 2. If, however, a fluid is brought from a state of motion to a state of rest by friction, as occurs, for example, at the surface of a flat plate that is oriented parallel to an air stream, the temperature of the fluid is raised to a value that may be different from that obtained by adiabatic compression. The ratio of the temperature rise due to friction to the temperature rise due to adiabatic compression is known as the recovery factor. The value of the recovery factor depends on whether the flow is laminar or turbulent. The recovery factor for flat plates in laminar flow has been shown theoretically in references 3, 4, and 5 to be very nearly equal to the square root of the Prandtl number. The Prandtl number of air at ordinary temperatures is about 0.75, and the recovery factor is about 0.87.

The measured value of the recovery factor has been reported in reference 6 to be 0.87 for flat plates and in reference 7 to be 0.91 for the laminar flow at 0.1 chord of an airfoil. Theoretical determinations of the recovery factor in turbulent flow have not been precise. (See reference 8.) The measured value reported in reference 6 is 0.90 for flat plates and in reference 7 is 0.95 for the turbulent flow at 0.7 chord of an airfoil. The value of the recovery factor has not been measured at high temperatures or at high supersonic Mach numbers. (Measurements at low supersonic Mach numbers have been reported in reference 7.) In the present paper a recovery factor of unity has been used.

Radiation

The third and fourth factors that affect surface temperatures are radiation and heat transfer by convection. Notwithstanding the fact that in supersonic flight the surface temperature (boundary-layer temperature) is much higher than the stream temperature, there is, of course, no transfer of heat between the surface and the stream provided that the surface is at the temperature to which the air in the boundary layer has been raised in being brought to rest. If, however, the surface loses heat by radiation and drops in temperature, there will be transfer of heat by convection from the air to the surface. It is obvious that under conditions of constant Mach number and altitude the processes of heat loss by radiation and heat gain by convection will come to equilibrium at a surface temperature that is lower than the surface temperature that would exist without radiation. The effects of radiation and convection must therefore be evaluated.

The rate at which energy is lost by a surface by radiation is

$$\frac{H}{S} = \epsilon\sigma T_s^4 \tag{10}$$

For the present calculations the following values of ϵ were used (compare reference 9, pp. 50-51 and table XIII, pp. 393-396):

$\epsilon = 0.2$ (corresponding approximately to oxidized aluminum at 1000° F and to polished iron at 100° to 500° F)

$\epsilon = 0.5$ (corresponding approximately to smooth sheet iron at 1900° F and oxidized rolled sheet steel at 100° F)

$\epsilon = 1.0$ (corresponding to a black body or perfect radiator)

For nearly all substances ϵ is a function of temperature. It will be shown, however, that, so long as the value of ϵ is not close to zero, the particular value that it has is not an important factor in determining airplane surface temperatures.

The rate of radiation H/S as given by equation (10) is shown as a function of surface temperature in figure 3.

Heat Transfer by Convection

The flow over the first part of a smooth flat plate is laminar. If the Reynolds number of the flow, based on the plate length, is sufficiently large, the region of laminar flow is followed by a region of transition, which in turn is followed by a region of turbulent flow. The equations for the heat-transfer coefficient in the laminar region are (reference 10, p. 238)

$$h_x = 0.33 \frac{k}{x} (Re)^{1/2} (Pr)^{1/3} \tag{11}$$

$$\bar{h} = 0.66 \frac{k}{x} (Re)^{1/2} (Pr)^{1/3} \tag{12}$$

In the turbulent region the equation for the local coefficient is (reference 10, p. 269)

$$h_x = 0.029 \frac{k}{x} (Re)^{0.8} Pr \tag{13}$$

and for the average coefficient is

$$\bar{h} = \frac{\int h_x \, dx}{\int dx} = 0.036 \frac{k}{x} (Re)^{0.8} Pr \tag{14}$$

Equation (12) is a theoretical equation that has been experimentally verified, and equation (14) is an empirical relation. These equations apply when the free-stream velocity is subsonic and when the temperature gradient (that is, the viscosity gradient) in the boundary layer is small but are assumed herein to apply in supersonic flow and in flow with large boundary-layer gradients.

Ordinarily, in order to compute the heat transfer from a flat plate, estimates are made of the Reynolds number of transition and the limits of the transition region. The value of h_x is then computed and plotted in the laminar and the turbulent regions, a reasonable transition curve is drawn, and an integration is performed to obtain the average heat-transfer coefficient for the entire plate. The extent, however, of the laminar region depends not only on the Reynolds number of the flow but also on the initial turbulence in the flow and on the viscosity gradient in the boundary layer. Measurements on an unheated flat plate by Van der Hegge Zijnen showed that transition occurred at a Reynolds number of about 300,000 with a given degree of initial turbulence and at a Reynolds number of 100,000 when a wire screen was used to increase the initial turbulence (reference 10, pp. 261-264). Measurements at the National Bureau of Standards showed transition at a Reynolds number of 1,100,000 with small initial turbulence and at 300,000 behind a wire screen (reference 10, pp. 264-265). The plotted data of Elias and of Fage and Falkner (reference 9, fig. 99, p. 206) would indicate transition on a heated flat plate at a Reynolds number of approximately 40,000 (if a transition curve were drawn between the laminar and the turbulent curves).

The airfoil to which the heat-transfer equations are applied is shown in figure 4. The conditions in region 2 between the shock wave and the airfoil surface, but outside the boundary layer, are given in terms of the conditions in region 1 ahead of the shock wave by the following equations (reference 11):

$$\frac{\rho_2}{\rho_1} = \frac{\tan \alpha}{\tan (\alpha - \beta)} \tag{15}$$

$$\frac{V_2}{V_1} = \frac{\cos \alpha}{\cos (\alpha - \beta)} \tag{16}$$

$$\frac{T_2}{T_1} = \frac{\left(\gamma M_1^2 \sin^2\alpha - \frac{\gamma - 1}{2}\right) \tan (\alpha - \beta)}{\frac{\gamma + 1}{2} \tan \alpha} \tag{17}$$

$$\cot \beta = \tan \alpha \left(\frac{\frac{\gamma + 1}{2} M_1^2}{M_1^2 \sin^2\alpha - 1} \right) \tag{18}$$

Conditions in region 1 depend on the Mach number and the altitude. The values of temperature and density that were used at the high altitudes considered herein are given in appendix A. The Reynolds numbers behind the shock wave, region 2, based on a length of 1 foot and a free-stream Mach number of 6 are shown in the following table:

Altitude (ft)	Reynolds number
50,000	9,600,000
100,000	880,000
150,000	81,000
200,000	7,400

The variation of Reynolds number with Mach number at a given altitude is not very large compared with the variation of Reynolds number with altitude at a given Mach number. The Reynolds number at a Mach number of 10 is approximately 7 times the Reynolds number at a Mach number of 2 at all the altitudes shown in the preceeding table.

The rate of heat transfer to the airfoil is

$$\frac{H}{S} = \bar{h}(T_t - T_s) \tag{19}$$

In applying equation (19), the following assumptions and simplifications were made:

(1) At altitudes of 50,000 and 100,000 feet, $\bar{h}$ is given by equation (14).

(2) At altitudes of 150,000 and 200,000 feet, $\bar{h}$ is given by equation (12).

(3) The k in equations (12) and (14) was based on the temperature in region 2, notwithstanding the large temperature gradient in the boundary layer.

(4) The Reynolds number used in computing $\bar{h}$ was based on density, velocity, and viscosity in region 2, notwithstanding the large viscosity gradient in the boundary layer.

(5) The Reynolds number was based on a 1-foot length.

The rates of heat transfer from the air to the airfoil, as given by equation (19) under these assumptions and simplifications, are shown in figure 5. The rate of heat transfer to the airfoil at a given Mach number decreases rapidly as the altitude is increased.

RESULTS AND DISCUSSION

At a given altitude and Mach number if thermal equilibrium has been reached and if there is no thermal conduction from the forward half of the airfoil (fig. 4) to some other part of the airplane, the equilibrium temperature of the forward part of the airfoil is the temperature at which the rate of heat transfer from the airfoil by radiation is equal to the rate of heat transfer to the airfoil by convection. In other words, equilibrium temperature is the surface temperature T_s at which H/S in figure 3 is equal to H/S in figure 5. This equilibrium surface temperature is shown in figure 6 as a function of free-stream Mach number. (The curve for $\epsilon = 0$ in figure 6 shows stagnation temperature.)

Figure 6 shows that the surface temperature can be much less than the stagnation temperature if the altitude of flight is sufficiently high. It can be seen, also, that, if the surface is in thermal equilibrium at a given temperature, the higher the altitude the greater the Mach number. An equilibrium surface temperature of 1500° F absolute is shown, for example, for $\epsilon = 0.5$, at a Mach number of 4 at 50,000 feet, of 4.4 at 100,000 feet, of 6.5 at 150,000 feet, and of 9.2 at 200,000 feet. At a given Mach number, therefore, the surface temperature is also shown to decrease as the altitude increases. At a Mach number of 8, for example, the surface temperature for $\epsilon = 0.5$ is 3800° F absolute at 50,000 feet and decreases to 1350° F absolute at 200,000 feet. Figure 6 shows clearly that the value of ϵ does not have much effect on the surface temperature so long as the value of ϵ is not close to zero.

The temperature of only the forward part (the hottest part) of the airfoil shown in figure 4 is considered herein. When the air stream expands in going from region 2 to region 3, its density is decreased more than its velocity is increased. The rate of convection from the air stream to the airfoil consequently is smaller and the equilibrium temperature is smaller for region 3 than for the forward part of the airfoil.

The effect of solar radiation on the temperature of the airfoil is not included in figure 6. The solar constant, which is the quantity of energy that impinges in unit time on unit area of a surface normal to the sun's rays and just outside the earth's atmosphere, is 0.116 Btu per second per square foot. (Actually, only about 70 percent of the initial solar radiation gets through the earth's atmosphere to sea level on a clear day when the sun is at the zenith.) For Mach numbers and altitudes for which the rate of gain of heat by the airfoil by convection and the rate of loss of heat by radiation are of the same order of magnitude as the solar constant, the temperature of the airfoil could be considerably affected by solar radiation. These conditions are met when the Mach number is small and the altitude is high. Figure 7 was prepared on the assumption that the airfoil absorbed radiation at the rate of 0.116ϵ Btu per second per square foot, in addition to receiving heat from the boundary layer by convection and losing heat by radiation. Comparison of figures 6(b) and 7 shows, therefore, the maximum effect that solar radiation can have on the equilibrium temperature of the airfoil.

Equilibrium temperatures at higher altitudes than 200,000 feet are not shown. The heat-transfer equations used for calculating equilibrium temperatures cannot be safely applied at altitudes greater than 200,000 feet inasmuch as the boundary-layer thickness is no longer large in comparison with the mean free path of the air molecules. (See appendix B.)

SUGGESTIONS FOR FUTURE RESEARCH

An interesting by-product of the investigation is the list of unsolved problems which made necessary a number of assumptions in obtaining the results. These problems include the following:

(1) Heat-transfer coefficient in supersonic flow

(2) Heat-transfer coefficient with large viscosity gradient

(3) Thermal conductivity and coefficient of viscosity of air at high temperatures

(4) Boundary-layer transition in supersonic flow

(5) Boundary-layer transition with large viscosity gradient

(6) Boundary-layer transition with heat transfer

(7) Recovery factor in supersonic flow

(8) Recovery factor with high boundary-layer temperatures

(9) Boundary-layer thickness in supersonic flow

(10) Drag, lift, and heat transfer at altitudes where mean free path is not of a lower order of magnitude than boundary-layer thickness

(11) Density and temperature at high altitudes

(12) Effect of heat-capacity lag on heat-transfer coefficient

CONCLUSIONS

The following conclusions based on calculations of surface temperatures of bodies in steady supersonic flight may be drawn:

1. The rate at which heat is transferred to a body from the air by means of convection decreases at constant Mach number as the altitude is increased.

2. Loss of heat from a body by means of radiation is an important factor in determining the temperature of the body if the altitude of flight is sufficiently great.

3. The temperature at which thermal equilibrium between convection and radiation is reached decreases, at constant Mach number, as the altitude is increased.

4. At sufficiently high altitudes and sufficiently high Mach numbers the surface temperature of a body is considerably less than the stagnation temperature of the air. At a Mach number of 8, for example, the stagnation temperature is 4800° F absolute and the equilibrium surface temperature for an emissivity of 0.5 is 3800° F absolute at 50,000 feet and decreases to 1350° F absolute at 200,000 feet.

5. The value of the emissivity of the surface does not have much effect on the surface temperature if the value of the emissivity is not close to zero.

6. Solar radiation has a large effect on the surface temperature only for low Mach numbers and high altitudes.

Langley Memorial Aeronautical Laboratory
National Advisory Committee for Aeronautics
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APPENDIX A

ATMOSPHERIC PROPERTIES AT HIGH ALTITUDES

The properties of the standard atmosphere are defined in reference 12 to an altitude of 65,000 feet. Altitude-pressure tables for altitudes to 80,000 feet are given in reference 13. In reference 13, at altitudes between 35,332 and 80,000 feet the temperature is taken to be 392° F absolute and the density is given by the following equation, which was obtained from Boyle's law and equation (8) of reference 13:

$$Y = 35332 + 48211.1 \log \frac{0.000727}{\rho_1} \tag{20}$$

The present investigation was simplified by the assumptions that the isothermal region of the atmosphere, in which the temperature is 392° F absolute, extends to an altitude of 200,000 feet and that the density in the isothermal region is given by equation (20). The following table gives the densities and pressures obtained under these assumptions:

Altitude (ft)	Density (slug/cu ft)	Pressure (in. Hg)
50,000	3.61×10^{-4}	3.43
100,000	3.31×10^{-5}	3.15×10^{-1}
150,000	3.04×10^{-6}	2.89×10^{-2}
200,000	2.79×10^{-7}	2.66×10^{-3}

Since the present paper was begun, the NACA Special Subcommittee on the Upper Atmosphere has adopted a resolution in which a tentative extension of the standard atmosphere is made. The extension is from the 65,000 feet altitude of reference 12 to 100,000 feet. The properties of this extended atmosphere are given in reference 14. The isothermal region is assumed to extend to 100,000 feet. The density and pressure shown in the preceding table at altitudes of 50,000 and 100,000 feet are accordingly the same as those given in reference 14 for those altitudes.

APPENDIX B

LIMITING ALTITUDE FOR FLUID-FLOW EQUATIONS

There is, or course, a limit to the altitude at which the usual equations describing fluid-flow phenomena apply. A boundary-layer phenomenon such as convective heat transfer depends on the free-path phenomena of viscosity and thermal conductivity. The usual equations that describe convective heat transfer can be expected to apply only when the mean free path of the air molecules is small in comparison with the thickness of the boundary layer. The mean free path of air molecules is given by the kinetic theory of gases as (reference 15)

$$\lambda = \frac{1}{\sqrt{2}\,\pi \nu D^2} \tag{21}$$

The thickness of the boundary layer, if taken as twice the displacement of the streamlines, is, for laminar flow (reference 16),

$$\delta = 3.4x(Re)^{-1/2} \tag{22}$$

The mean free path in region 2 of figure 4 between the shock wave and the airfoil surface (but outside the boundary layer) and the calculated thickness of the boundary layer on the airfoil 1/2 foot from the leading edge are shown in the following table. The values are for a Mach number of 6. (The values of λ and δ vary by a maximum factor of about 3 between a Mach number of 2 and a Mach number of 10.)

Altitude (ft)	Mean free path (in.)	Boundary-layer thickness (in.)
50,000	1.7×10^{-5}	9.3×10^{-3}
100,000	1.9×10^{-4}	3.1×10^{-2}
150,000	2.0×10^{-3}	1.0×10^{-1}
200,000	2.2×10^{-2}	3.0×10^{-1}

According to the data of the preceding table, the equations for heat-transfer coefficient given in the present paper cannot be safely applied at altitudes exceeding 200,000 feet.

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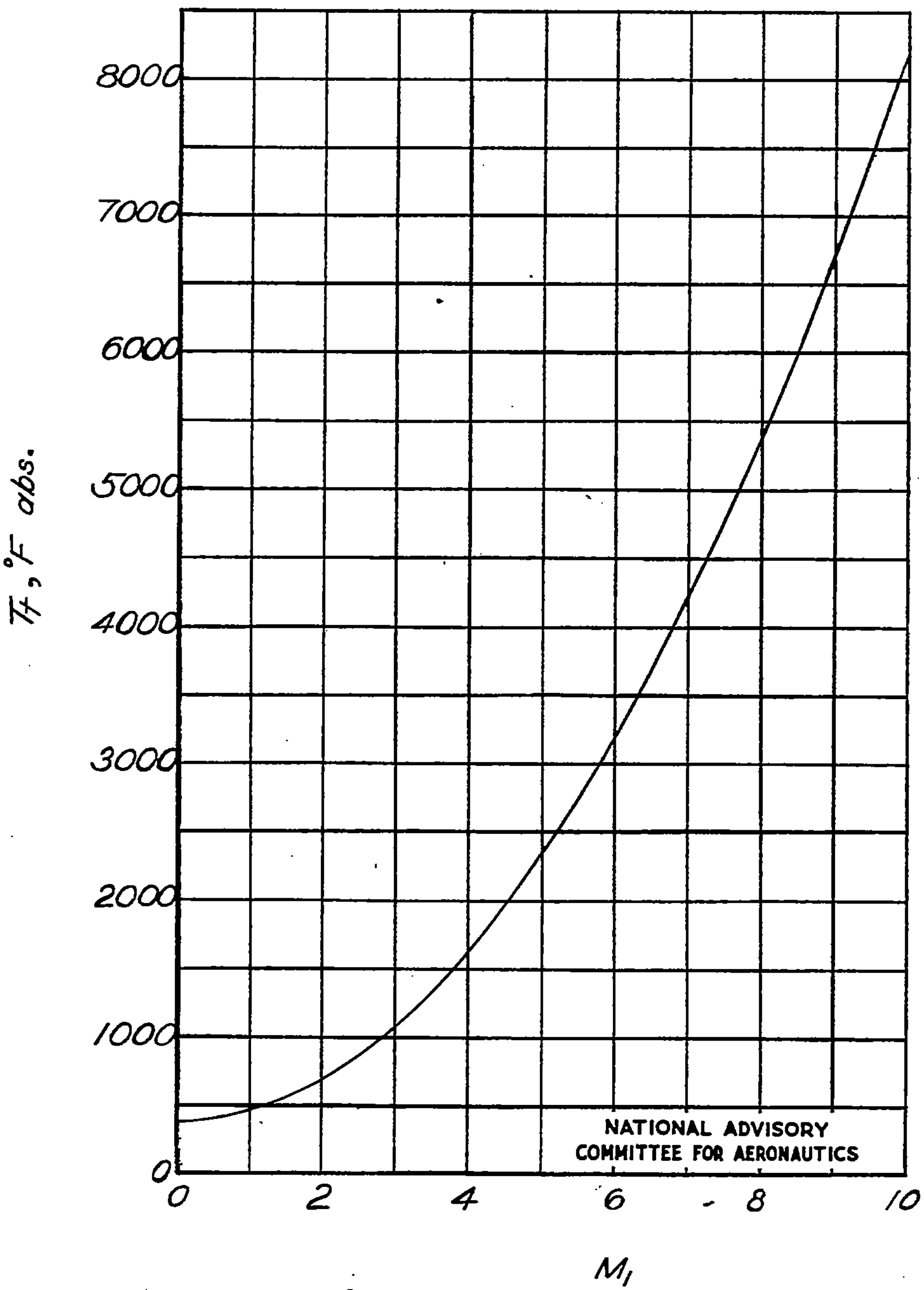


Figure 1.– Stagnation temperature as a function of free-stream Mach number. Free-stream temperature, 392 °F abs.; $\gamma = 1.4$.

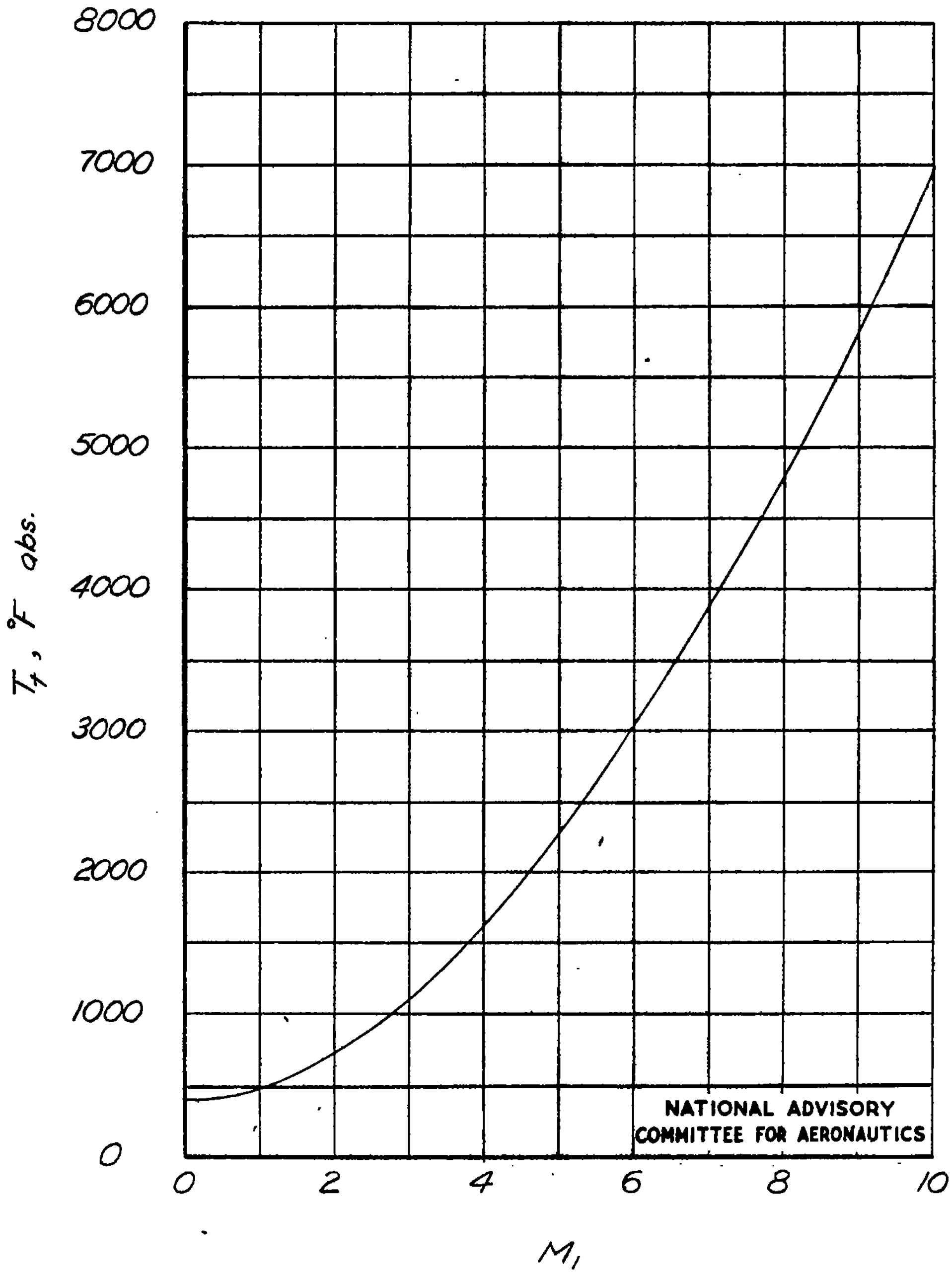
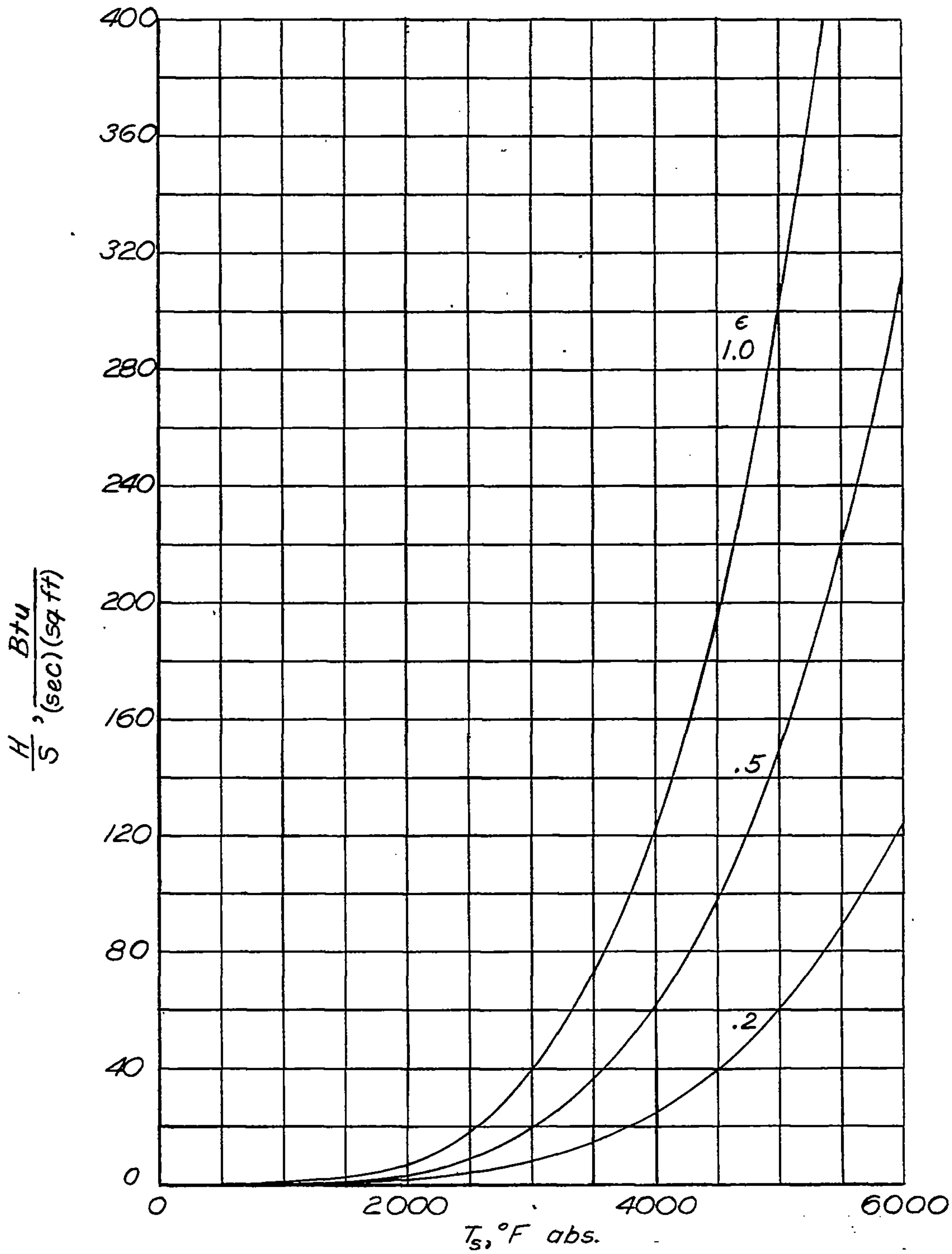


Figure 2. – Stagnation temperature as a function of free-stream Mach number. Free-stream temperature, 392 °F abs.; γ varies with temperature.



(a) T_s from 0° to 6000° F abs.

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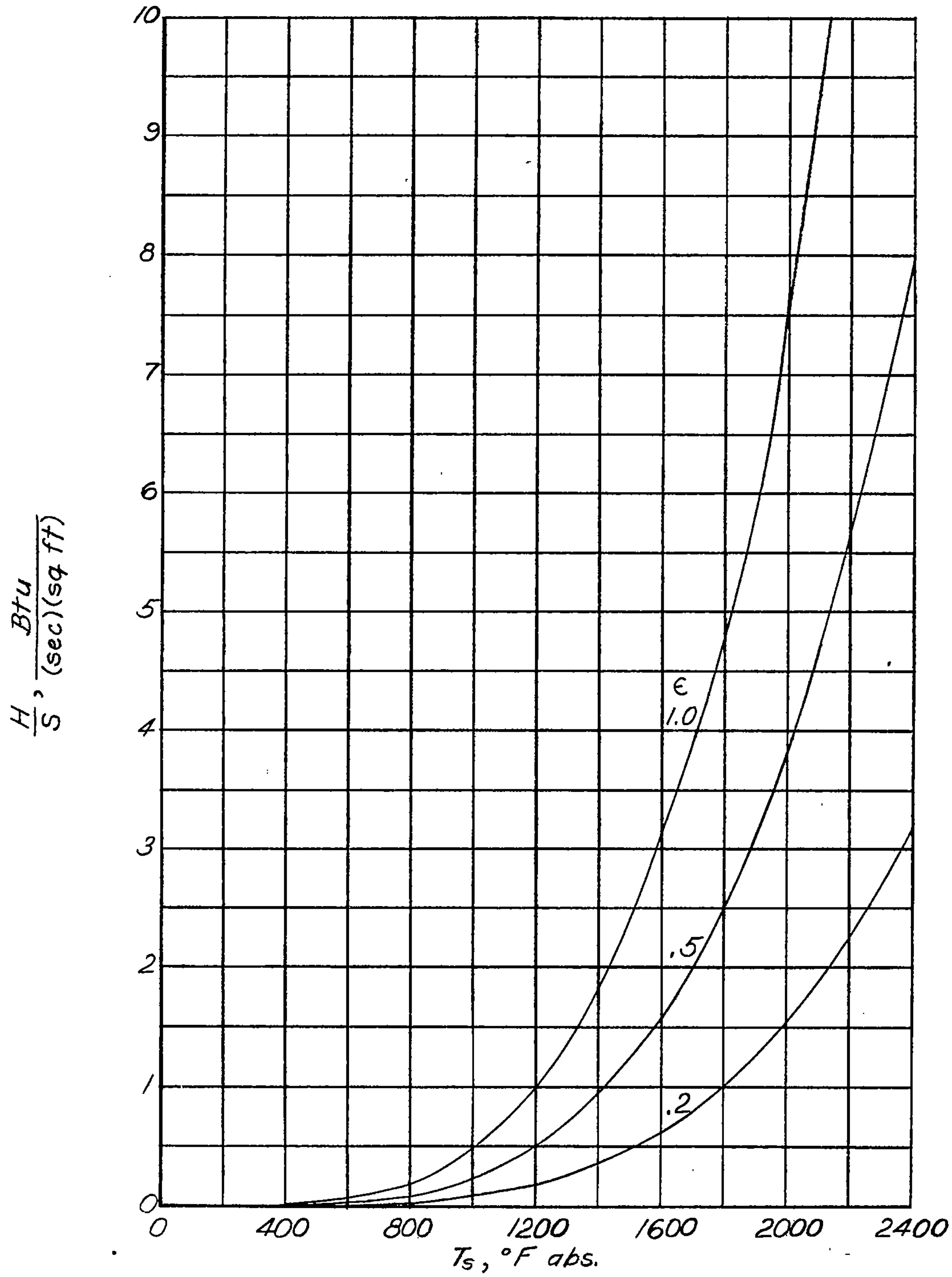
Figure 3.- Rate of heat emission by radiation as a function of surface temperature.



(b) T_s from 0° to 2400° F abs.

Figure 3.- Concluded.

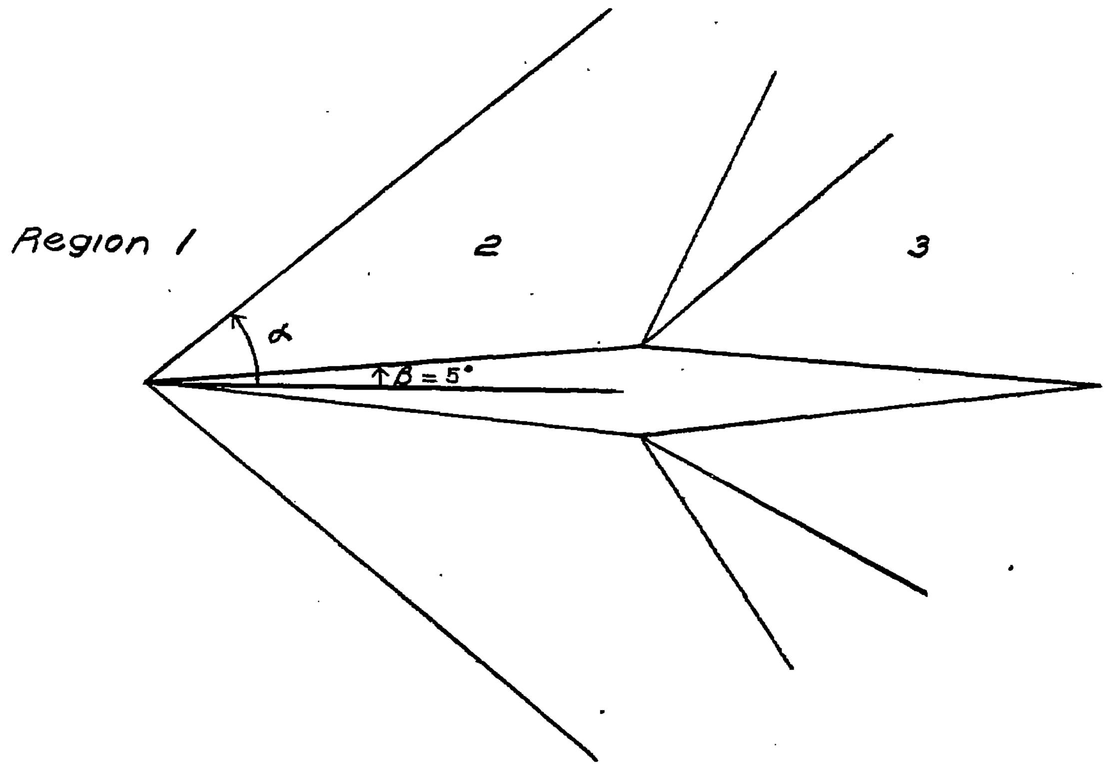


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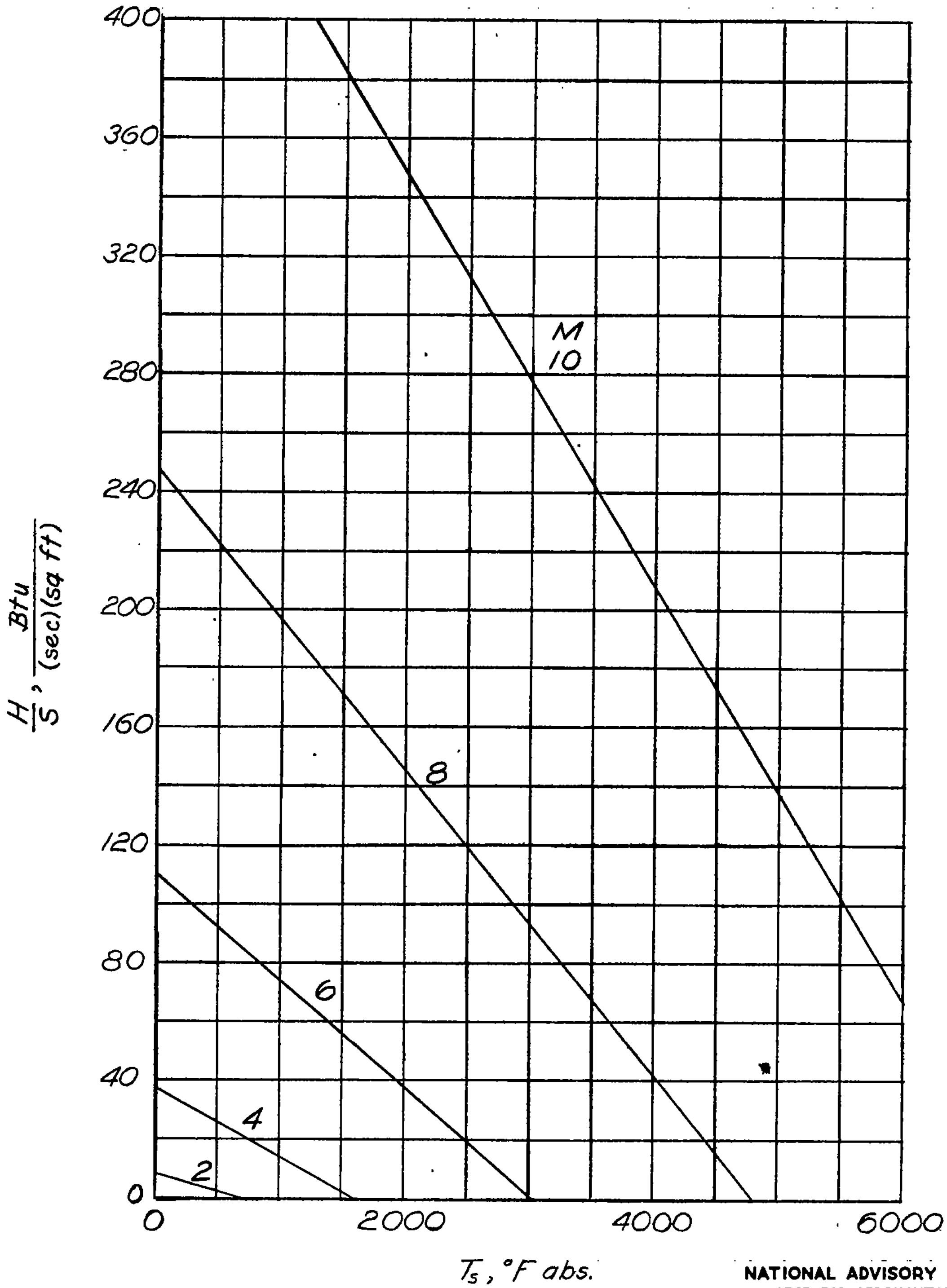
Figure 4.- Airfoil and designation of regions and angles.



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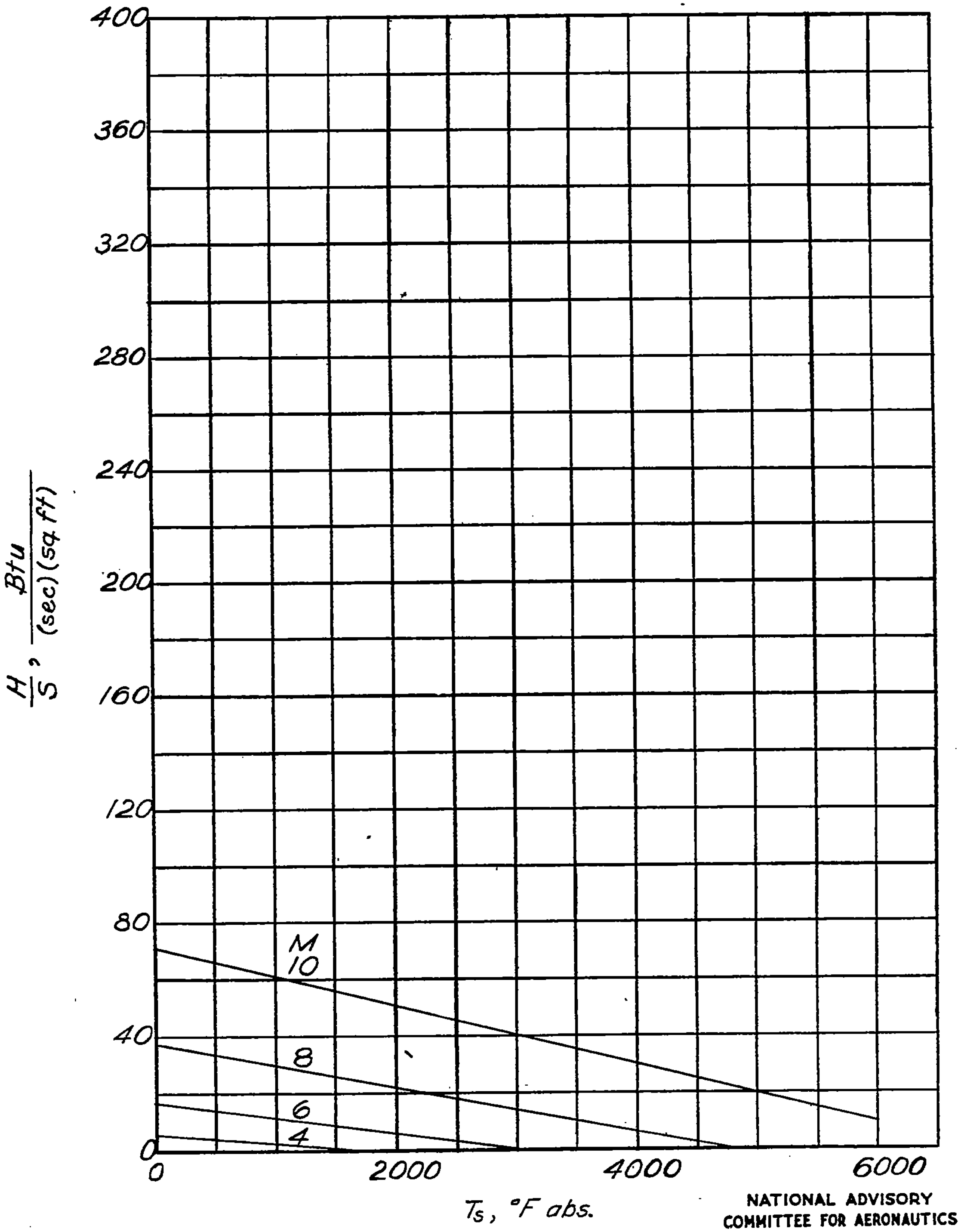
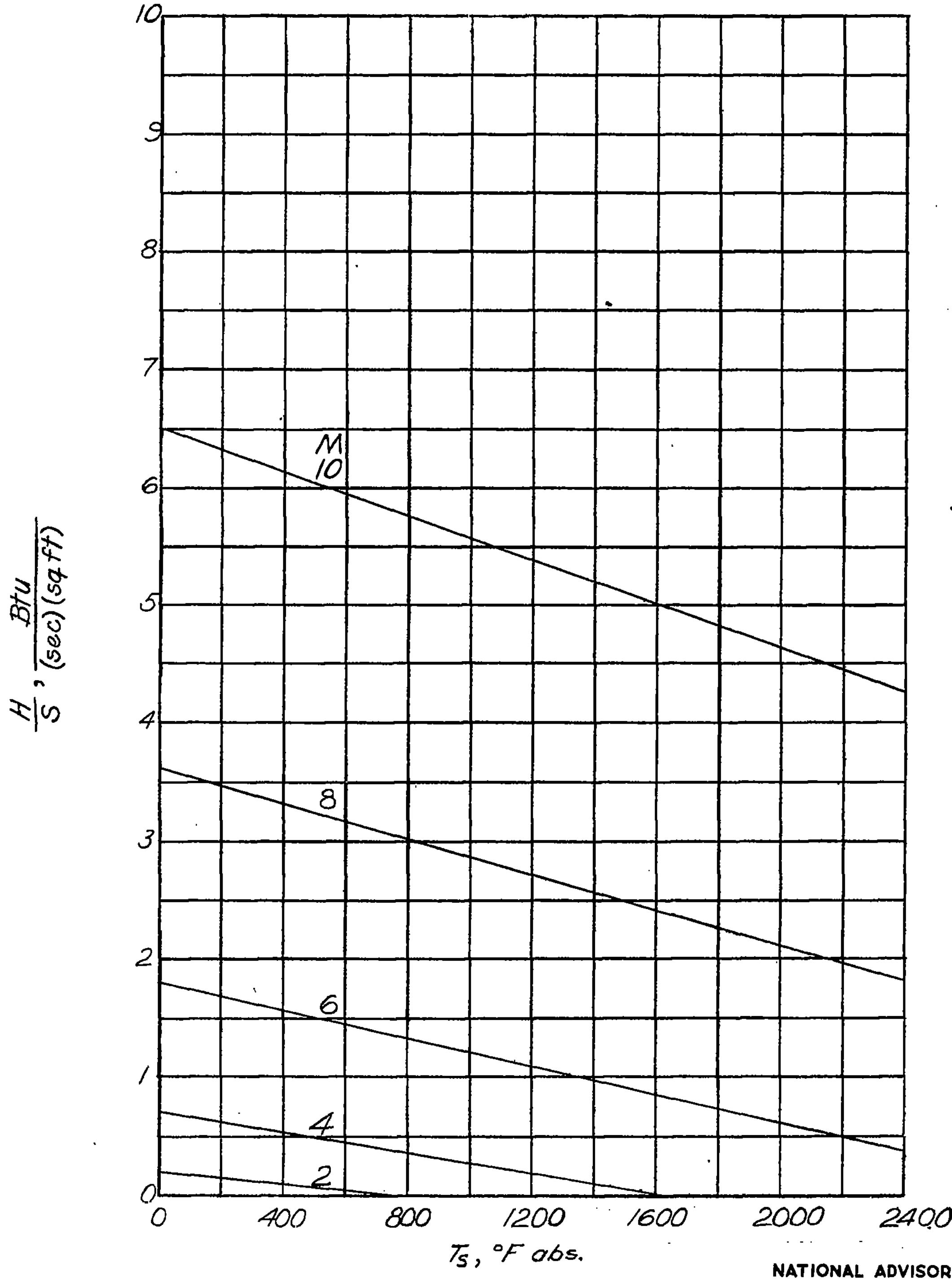
(a) Altitude, 50,000 feet.

Figure 5. - Rate of heat gain by convection as a function of surface temperature.



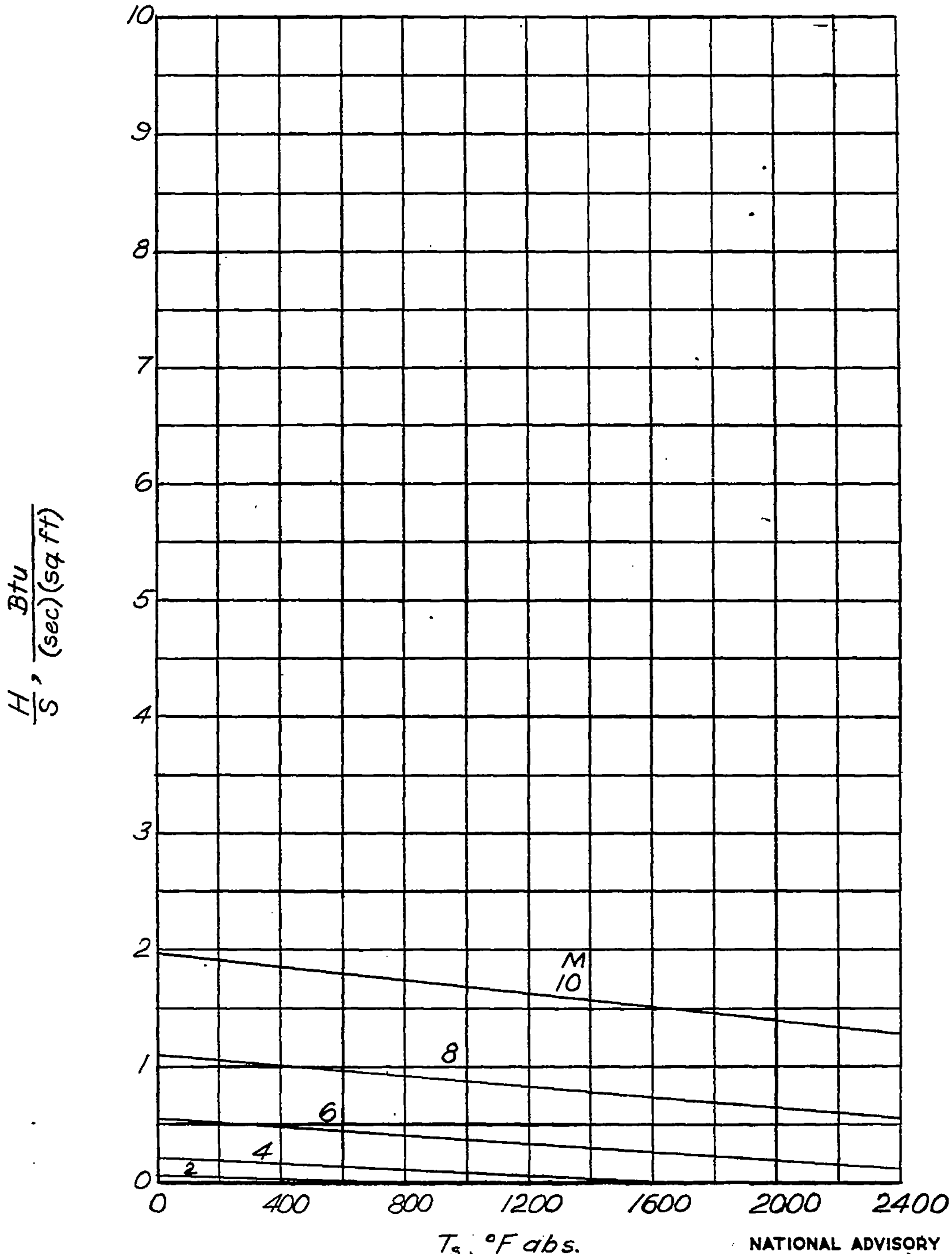
(b) Altitude, 100,000 feet.

Figure 5.– Continued.



(c) Altitude, 150,000 feet.

Figure 5. - Continued.



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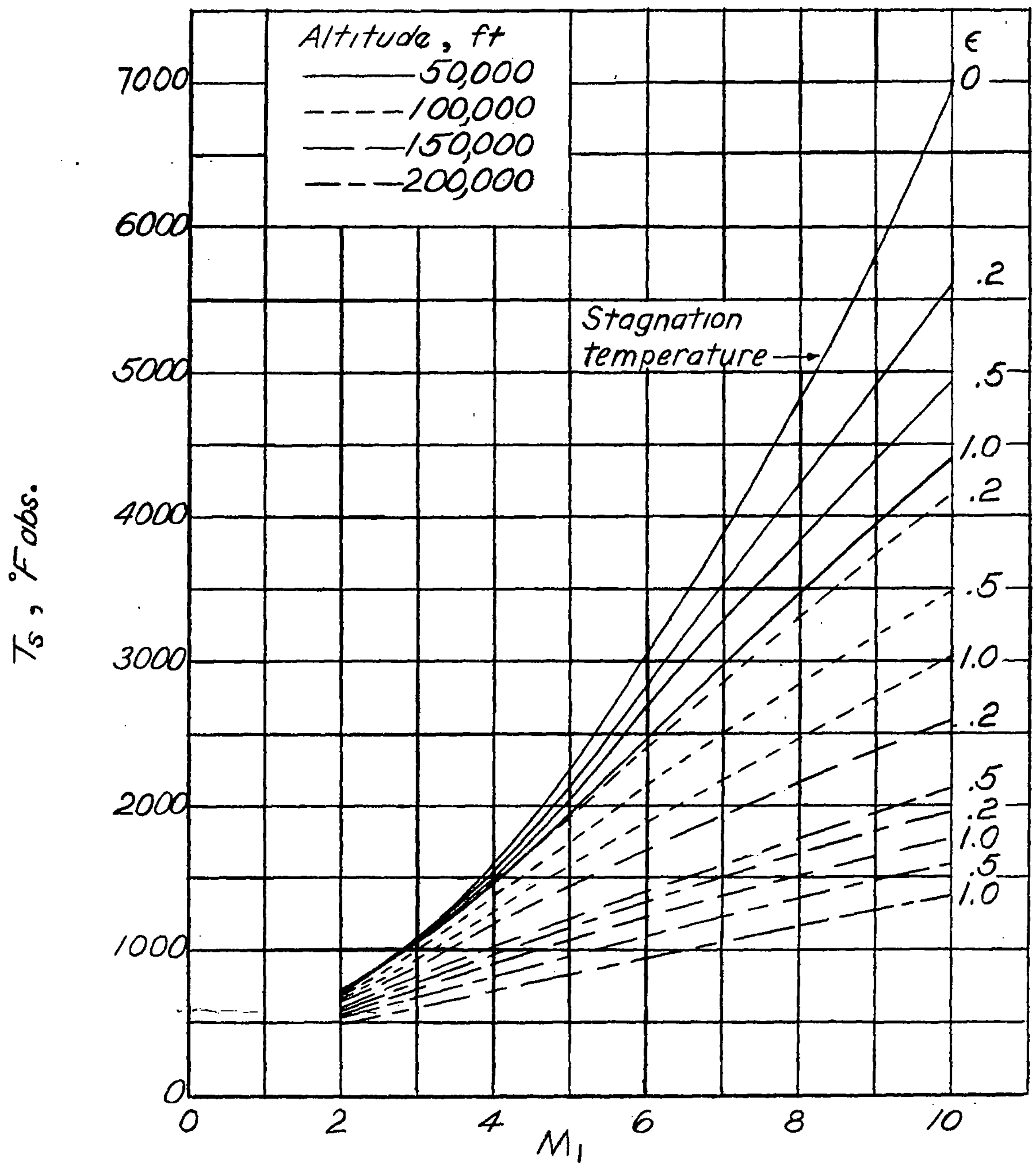
(d) Altitude, 200,000 feet.

Figure 5. - Concluded.



(a) M_1 from 2 to 10.

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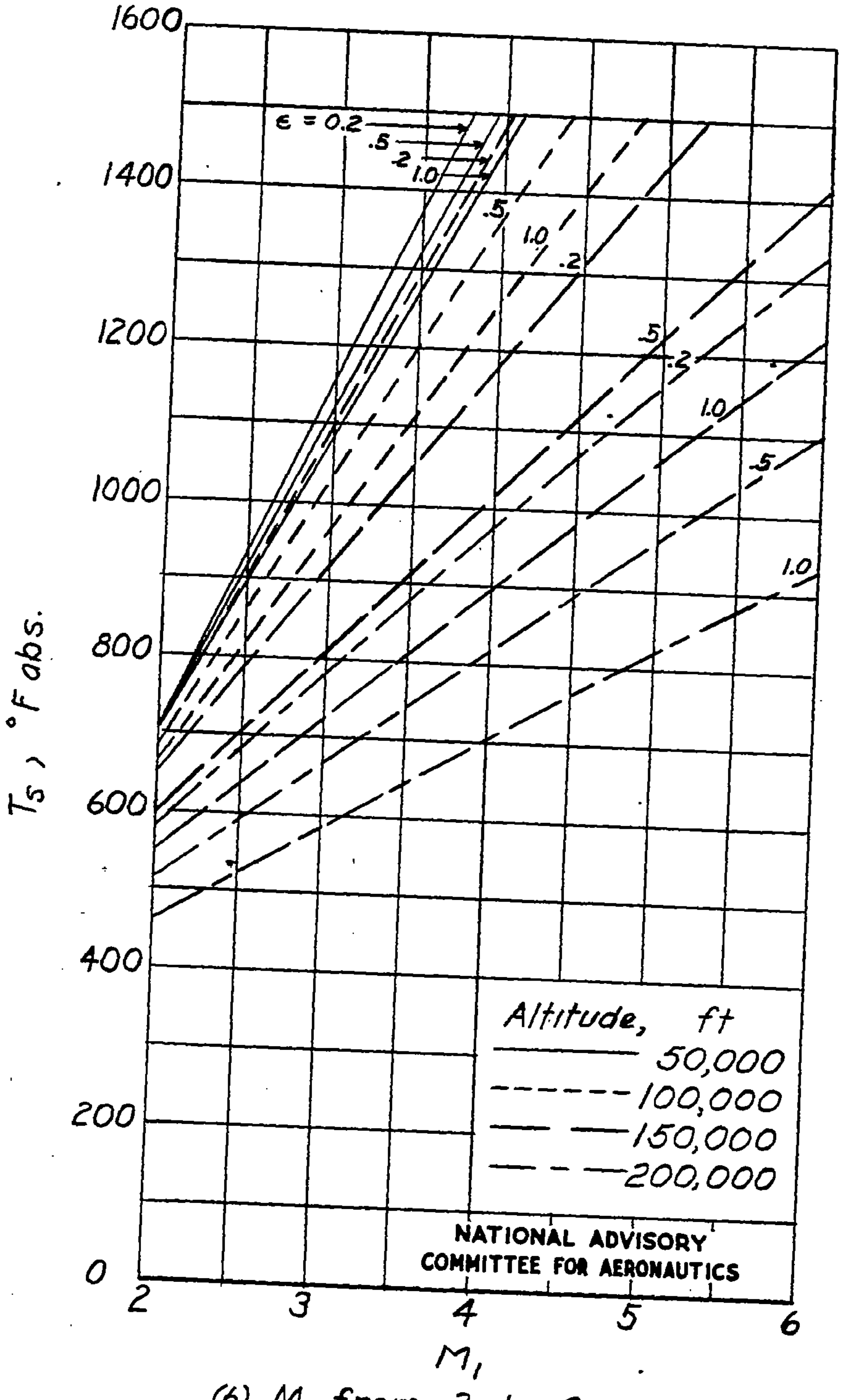
Figure 6.- Equilibrium surface temperature as a function of free-stream Mach number.



(b) M_1 from 2 to 6.

Figure 6.- Concluded.

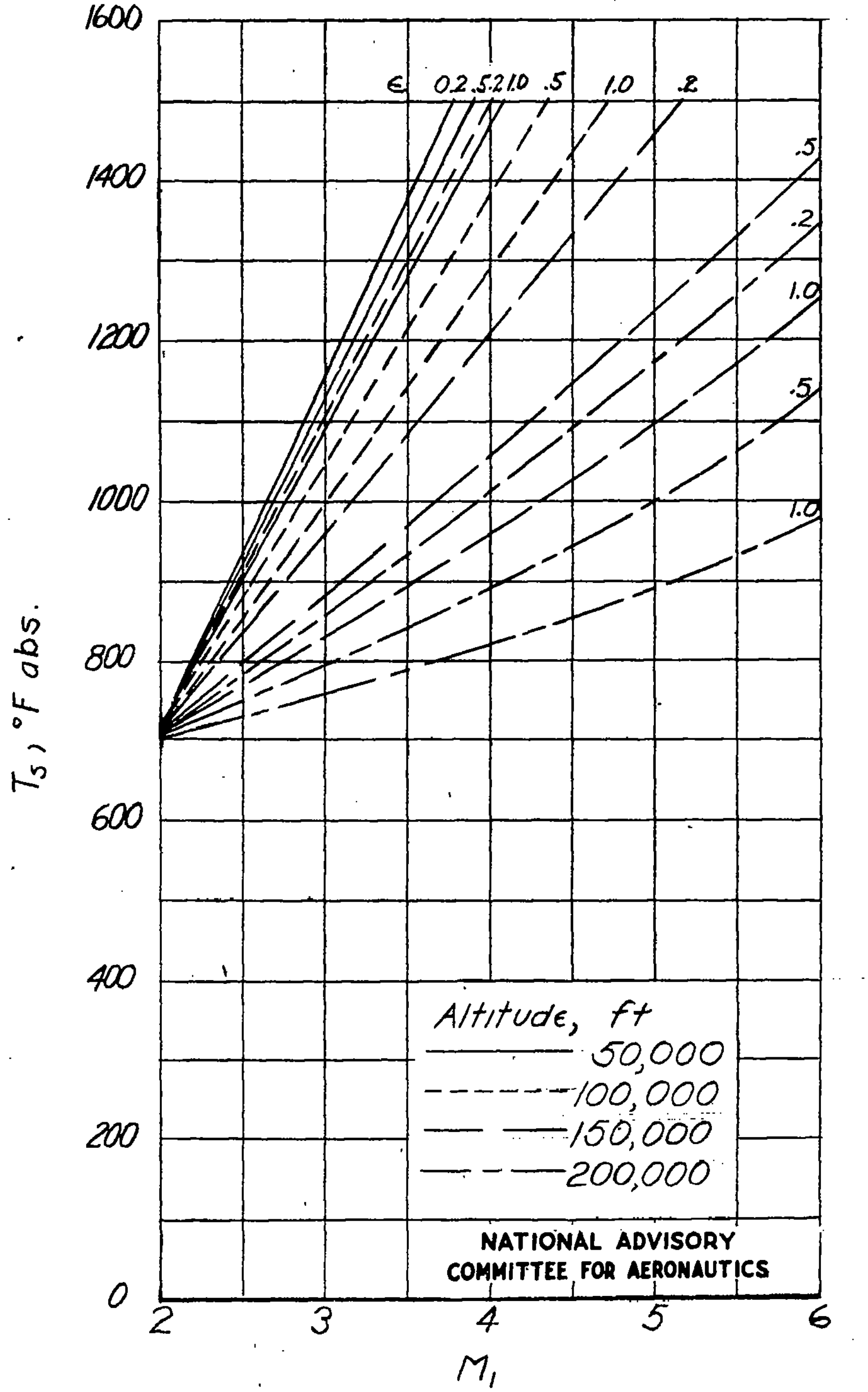


Figure 7.- Equilibrium surface temperature as a function of free-stream Mach number. Effect of solar radiation included.