CHAPTER 5
THE TURBOFAN CYCLE

5.1 TURBOFAN THRUST

The figure below illustrates two generic turbofan engine designs.

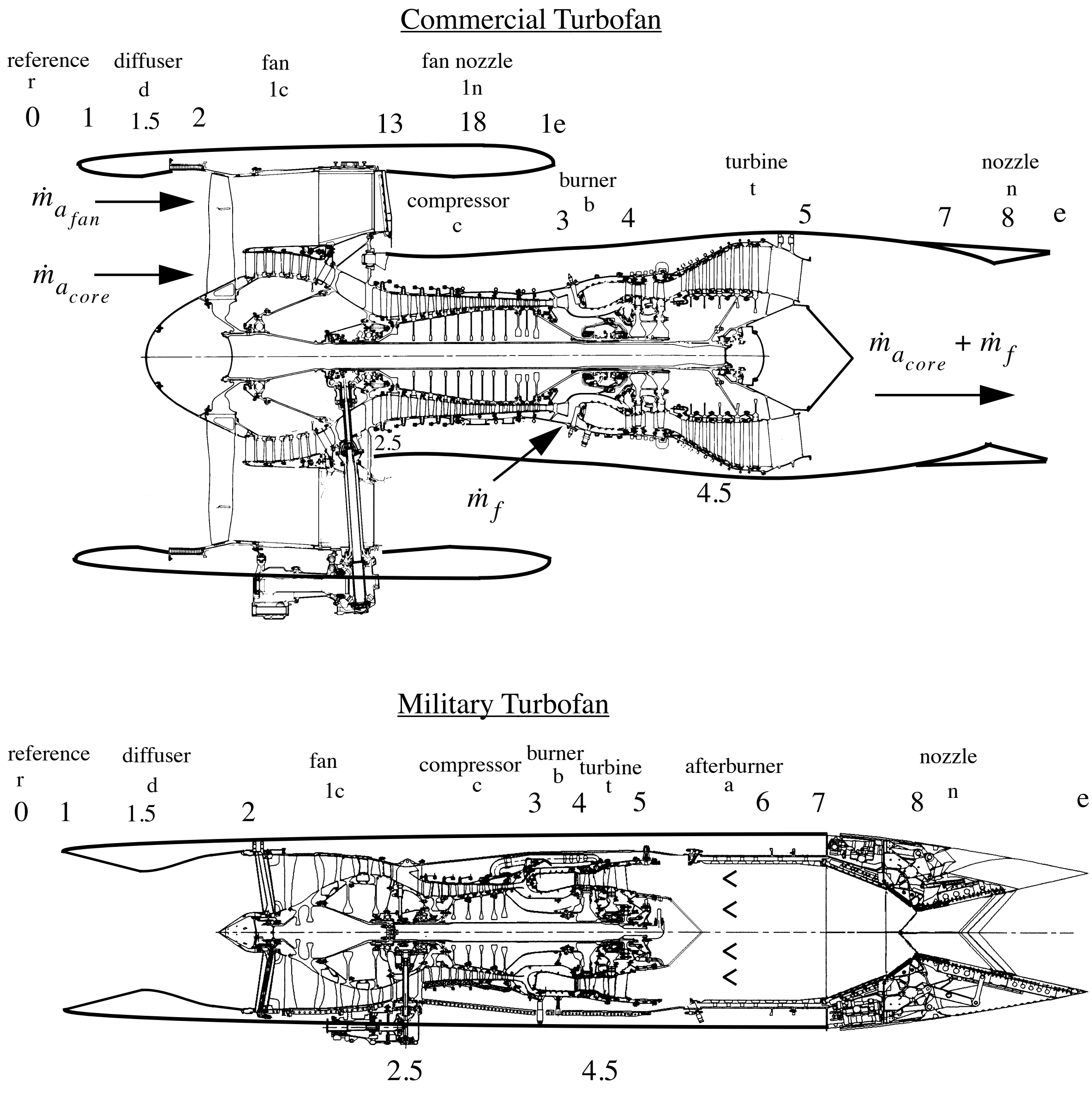


Figure 5.1   Engine numbering and component notation

The upper figure shows a modern high bypass ratio engine designed for long distance cruise at subsonic Mach numbers around 0.83 typical of a commercial aircraft. The fan utilizes a single stage composed of a large diameter fan (rotor) with wide chord blades followed by a single nozzle stage (stator). The bypass ratio is 5.8 and the fan pressure ratio is 1.9. The lower figure shows a military turbofan designed for high performance at supersonic Mach numbers in the range of 1.1 to 1.5. The fan on this engine has three stages with an overall pressure ratio of about 6 and a bypass ratio of only about 0.6. One of the goals of this chapter is to understand why these engines look so different in terms the differences in flight condition for which they are designed. In this context we will begin to appreciate that the thermodynamic and gasdynamic analysis of these engines defines a continuum of cycles as a function of Mach number. We had a glimpse of this when we determined that the maximum thrust turbojet is characterized by

$$\tau_{c_{max\ thrust}} = \frac{\sqrt{\tau_\lambda}}{\tau_r}. \tag{5.1}$$

For fixed turbine inlet temperature and altitude, as the Mach number increases the optimum compression decreases and at some point it becomes desirable to convert the turbojet to a ramjet. We will see a similar kind of trend emerge for the turbofan where it replaces the turbojet as the optimum cycle for lower Mach numbers. Superimposed on all this is a technology trend where, with better materials and cooling schemes, the allowable turbine inlet temperature increases. This tends to lead to an optimum cycle with higher compression and higher bypass ratio at a given Mach number.

The thrust equation for the turbofan is similar to the usual relation except that it includes the thrust produced by the fan.

$$T = \dot{m}_{a_{core}}(U_e - U_0) + \dot{m}_{a_{fan}}(U_{1e} - U_0) + \dot{m}_f U_e + (P_e - P_0)A_e + (P_{1e} - P_0)A_{1e} \tag{5.2}$$

The total air mass flow is

$$\dot{m}_a = \dot{m}_{a_{core}} + \dot{m}_{a_{fan}}. \tag{5.3}$$

The fuel/air ratio is defined in terms of the total air mass flow.

$$f = \frac{\dot{m}_f}{\dot{m}_a}. \tag{5.4}$$

The bypass fraction is defined as

$$B = \frac{\dot{m}_{a_{fan}}}{\dot{m}_{a_{fan}} + \dot{m}_{a_{core}}} \tag{5.5}$$

and the bypass ratio is

$$\beta = \frac{\dot{m}_{a_{fan}}}{\dot{m}_{a_{core}}}. \tag{5.6}$$

Note that

$$\beta = \frac{B}{1-B} \qquad B = \frac{\beta}{1+\beta}. \tag{5.7}$$

5.2 THE IDEAL TURBOFAN CYCLE

In the ideal cycle we will make the usual assumption of isentropic flow in the inlet, fan, compressor, turbine and fan and core nozzles as well as the assumption of low Mach number heat addition in the burner. The fan and core nozzles are assumed to be fully expanded. The assumptions are

$$P_{1e} = P_0 \qquad P_e = P_0 \tag{5.8}$$

$$\pi_d = 1 \qquad \pi_b = 1 \qquad \pi_n = 1 \qquad \pi_{n1} = 1 \tag{5.9}$$

$$\pi_c = \tau_c^{\frac{\gamma}{\gamma-1}} \qquad \pi_{c1} = \tau_{c1}^{\frac{\gamma}{\gamma-1}} \qquad \pi_t = \tau_t^{\frac{\gamma}{\gamma-1}}. \tag{5.10}$$

For a fully expanded exhaust the normalized thrust is

$$\frac{T}{\dot{m}_a a_0} = M_0\left\{(1-B+f)\left(\frac{U_e}{U_0}-1\right) + B\left(\frac{U_{1e}}{U_0}-1\right) + f\right\} \tag{5.11}$$

or, in terms of the bypass ratio with $f \ll 1$

$$\frac{T}{\dot{m}_a a_0} = M_0\left\{\left(\frac{1}{1+\beta}\right)\left(\frac{U_e}{U_0}-1\right) + \left(\frac{\beta}{1+\beta}\right)\left(\frac{U_{1e}}{U_0}-1\right)\right\}. \tag{5.12}$$

5.2.1 THE FAN BYPASS STREAM

First work out the velocity ratio for the fan stream

$$\frac{U_{1e}}{U_0} = \frac{M_{1e}}{M_0}\sqrt{\frac{T_{1e}}{T_0}}. \tag{5.13}$$

The exit Mach number is determined from the stagnation pressure.

$$P_{t1e} = P_0\pi_r\pi_{c1} = P_{1e}\left(1+\frac{\gamma-1}{2}M_{1e}^2\right)^{\frac{\gamma}{\gamma-1}}. \tag{5.14}$$

Since the nozzle is fully expanded and the fan is assumed to behave isentropically, we can write

$$\tau_r\tau_{c1} = 1+\frac{\gamma-1}{2}M_{1e}^2 \tag{5.15}$$

therefore

$$\frac{M_{1e}^2}{M_0^2} = \frac{\tau_r \tau_{c1} - 1}{\tau_r - 1}. \tag{5.16}$$

The exit temperature is determined from the stagnation temperature.

$$T_{te} = T_0 \tau_r \tau_{c1} = T_{1e}\left(1 + \frac{\gamma - 1}{2} M_{1e}^2\right). \tag{5.17}$$

Noting (5.15) we can conclude that for the ideal fan

$$T_{1e} = T_0. \tag{5.18}$$

The exit static temperature is equal to the ambient static temperature. The velocity ratio of the fan stream is

$$\left(\frac{U_{1e}}{U_0}\right)^2 = \frac{\tau_r \tau_{c1} - 1}{\tau_r - 1}. \tag{5.19}$$

5.2.2 THE CORE STREAM

The velocity ratio across the core is

$$\left(\frac{U_e}{U_0}\right)^2 = \left(\frac{M_e}{M_0}\right)^2 \frac{T_e}{T_0}. \tag{5.20}$$

The analysis of the stagnation pressure and temperature is exactly the same as for the ideal turbojet.

$$P_{te} = P_0 \pi_r \pi_c \pi_t = P_e\left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{\frac{\gamma}{\gamma - 1}}. \tag{5.21}$$

Since the nozzle is fully expanded and the compressor and turbine operate ideally the Mach number ratio is

$$\frac{M_e^2}{M_0^2} = \left(\frac{\tau_r \tau_c \tau_t - 1}{\tau_r - 1}\right). \tag{5.22}$$

The temperature ratio is also determined in the same way in terms of component temperature parameters

$$T_{te} = T_0 \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n. \tag{5.23}$$

In the ideal turbofan we assume that the diffuser and nozzle flows are adiabatic and so

$$T_{te} = T_0\tau_r\tau_c\tau_b\tau_t = T_e\left(1 + \frac{\gamma - 1}{2}M_e^2\right) = T_e\tau_r\tau_c\tau_t \tag{5.24}$$

from which is determined

$$\frac{T_e}{T_0} = \frac{\tau_\lambda}{\tau_r\tau_c}. \tag{5.25}$$

The velocity ratio across the core is

$$\left(\frac{U_e}{U_0}\right)^2 = \left(\frac{\tau_r\tau_c\tau_t - 1}{\tau_r - 1}\right)\frac{\tau_\lambda}{\tau_r\tau_c}. \tag{5.26}$$

5.2.3 Turbine - compressor - fan matching

The work taken out of the flow by the high and low pressure turbine is used to drive both the compressor and the fan.

$$(\dot{m}_{a_{core}} + \dot{m}_f)(h_{t4} - h_{t5}) = \dot{m}_{a_{core}}(h_{t3} - h_{t2}) + \dot{m}_{a_{fan}}(h_{t13} - h_{t2}) \tag{5.27}$$

Divide (5.27) by $\dot{m}_a C_p T_0$ and rearrange. The work matching condition for a turbofan is

$$\tau_t = 1 - \frac{\tau_r}{\tau_\lambda}\left\{\frac{1 - B}{1 - B + f}(\tau_c - 1) + \frac{B}{1 - B + f}(\tau_{c1} - 1)\right\}. \tag{5.28}$$

The approximation $f \ll 1$ is generally a pretty good one for a turbofan. Using this approximation the work matching condition becomes

$$\tau_t = 1 - \frac{\tau_r}{\tau_\lambda}\{(\tau_c - 1) + \beta(\tau_{c1} - 1)\} \tag{5.29}$$

where the bypass ratio β appears for the first time. If the bypass ratio goes to zero the matching condition reduces to the usual turbojet formula.

5.2.4 Fuel/air ratio

The fuel/air ratio is determined from the energy balance across the burner.

$$\dot{m}_f(h_f - h_{t4}) = \dot{m}_{a_{core}}(h_{t4} - h_{t3}). \tag{5.30}$$

Divide (5.30) by $\dot{m}_a C_p T_0$ and rearrange. The result is

$$f = \left(\frac{1}{1+\beta}\right)\frac{\tau_\lambda - \tau_r\tau_c}{\tau_f - \tau_\lambda} \quad . \tag{5.31}$$

5.3 MAXIMUM SPECIFIC IMPULSE IDEAL TURBOFAN

The specific impulse is

$$\frac{I_{sp}g}{a_0} = \left(\frac{T}{\dot{m}_f g}\right)\left(\frac{g}{a_0}\right) = \left(\frac{T}{\dot{m}_a a_0}\right)\left(\frac{1}{f}\right). \tag{5.32}$$

Substitute (5.12) and (5.31) into (5.32). The result is

$$\frac{I_{sp}g}{a_0} = M_0\left(\frac{\tau_f - \tau_\lambda}{\tau_\lambda - \tau_r\tau_c}\right)\left\{\left(\frac{U_e}{U_0} - 1\right) + \beta\left(\frac{U_{1e}}{U_0} - 1\right)\right\}. \tag{5.33}$$

The question is: what value of β maximizes the specific impulse? Differentiate (5.33) with respect to β and note that β appears in (5.29). The result is

$$\frac{\partial}{\partial\beta}\left(\frac{I_{sp}g}{a_0}\right) = \frac{\partial}{\partial\beta}\left(\frac{U_e}{U_0}\right) + \left(\frac{U_{1e}}{U_0} - 1\right) = 0\,. \tag{5.34}$$

We can write (5.34) as

$$\frac{1}{2(U_e/U_0)}\frac{\partial}{\partial\beta}\left(\frac{U_e^2}{U_0^2}\right) + \left(\frac{U_{1e}}{U_0} - 1\right) = 0 \tag{5.35}$$

or

$$\frac{1}{2(U_e/U_0)}\left(\frac{\tau_\lambda}{\tau_r - 1}\right)\frac{\partial\tau_t}{\partial\beta} = -\left(\frac{U_{1e}}{U_0} - 1\right). \tag{5.36}$$

This becomes

$$\frac{1}{2(U_e/U_0)}\left(\frac{\tau_r(\tau_{c1} - 1)}{\tau_r - 1}\right) = \left(\frac{U_{1e}}{U_0} - 1\right). \tag{5.37}$$

From (5.19),the expression in parentheses on the left side of (5.37) is

$$\frac{1}{2(U_e/U_0)}\left(\left(\frac{U_{1e}}{U_0}\right)^2 - 1\right) = \left(\frac{U_{1e}}{U_0} - 1\right). \tag{5.38}$$

Factor the left side of (5.38) and cancel common factors. The velocity condition for a maximum impulse ideal turbofan is

$$\left(\frac{U_{1e}}{U_0} - 1\right) = 2\left(\frac{U_e}{U_0} - 1\right). \tag{5.39}$$

According to this result for an ideal turbofan one would want to design the turbine such that the velocity increment across the fan was twice that across the core in order to achieve maximum specific impulse. Recall that U_e/U_0 depends on β through (5.29) (and weakly though (5.31) which we neglect). The value of β that produces the condition (5.39) corresponding to the maximum impulse ideal turbofan is

$$\beta_{max\ impulse\ ideal\ turbofan} = \frac{1}{(\tau_{c1} - 1)} \times$$

$$\left\{\left(\frac{\tau_\lambda}{\tau_r \tau_c} - 1\right)(\tau_c - 1) + \frac{\tau_\lambda}{\tau_r^2 \tau_c}(\tau_r - 1) - \frac{1}{4}\left(\frac{\tau_r - 1}{\tau_r}\right)\left(\left(\frac{\tau_r \tau_{c1} - 1}{\tau_r - 1}\right)^{1/2} + 1\right)\right\}. \tag{5.40}$$

The figure below shows how the optimum bypass ratio (5.40) varies with flight Mach number for a given set of engine parameters.

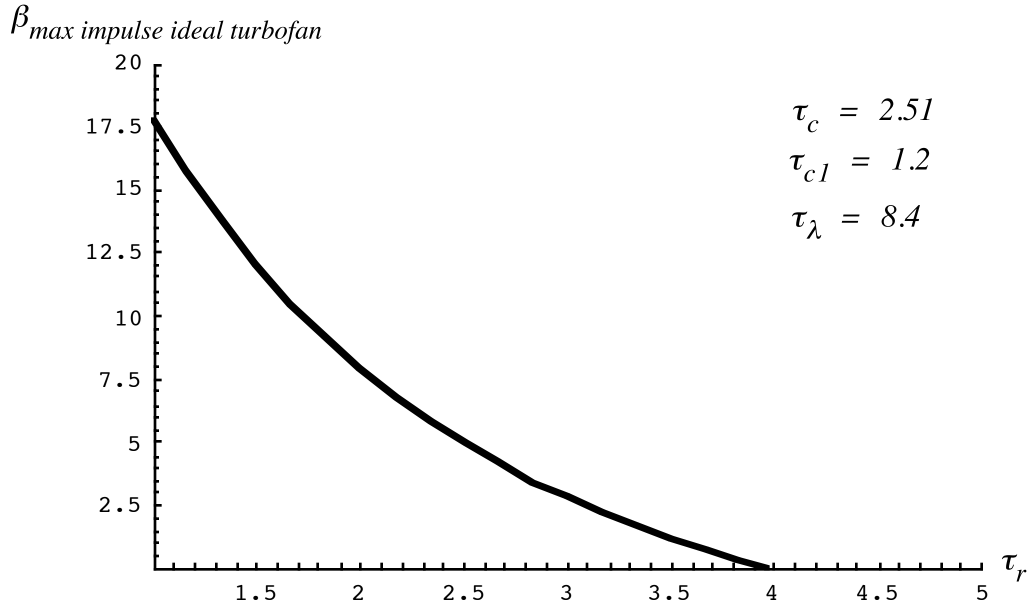


Figure 5.2 Ideal turbofan bypass ratio for maximum specific impulse as a function of Mach number.

It is clear from this figure that as the Mach number increases the optimum bypass ratio decreases until a point is reached where one would like to get rid of the fan altogether and convert the engine to a turbojet. For the ideal cycle the turbojet limit occurs at an unrealistically high Mach number of approximately 3.9. Non-ideal component behavior reduces this Mach number considerably.

The next figure provides another cut on this issue. Here the optimum bypass ratio is plotted versus the fan temperature (or pressure) ratio. Several curves are shown for increasing Mach number.

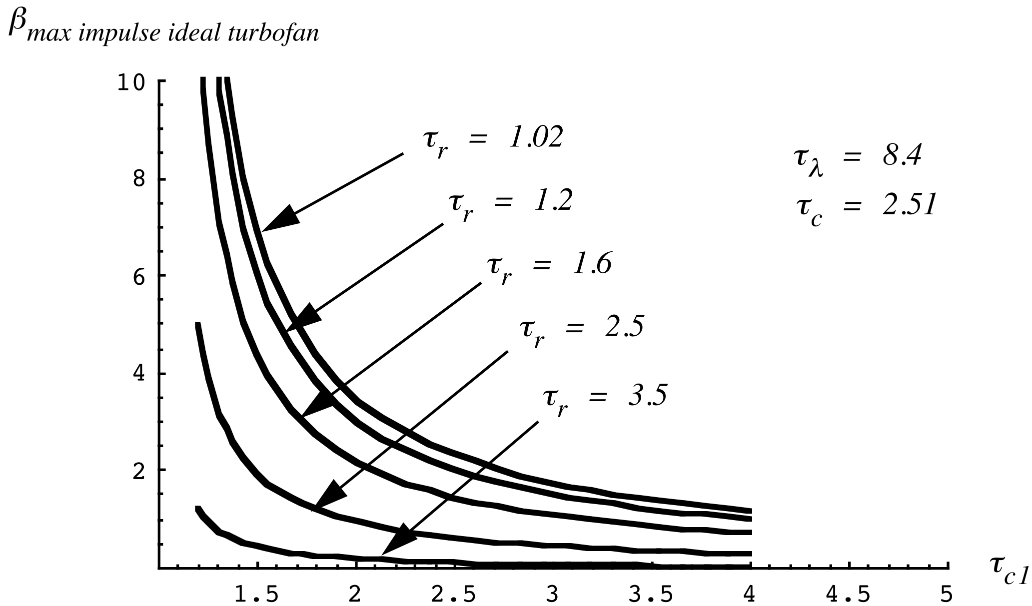


Figure 5.3 Ideal turbofan bypass ratio for maximum specific impulse as a function of fan temperature ratio. Plot shown for several Mach numbers

It is clear that increasing the fan pressure ratio leads to an optimum at a lower bypass ratio. The curves all seem to allow for optimum systems at very low fan pressure ratios and high bypass ratios. This is an artifact of the assumptions underlying the ideal turbofan. As soon as non-ideal effects are included the low fan pressure ratio solutions reduce to much lower bypass ratios. To see this we will compute several non-ideal cases.

5.4 TURBOFAN THERMAL EFFICIENCY

Recall the definition of thermal efficiency from Chapter 2.

$$\eta_{th} = \frac{Power\ to\ the\ vehicle + \dfrac{\Delta\ kinetic\ energy\ of\ air}{second} + \dfrac{\Delta\ kinetic\ energy\ of\ fuel}{second}}{\dot{m}_f h_f} \tag{5.41}$$

For a turbofan with a core and bypass stream the thermal efficiency is

$$\eta_{th} = \frac{TU_0 + \left[\dfrac{\dot{m}_{a_{core}}(U_e - U_0)^2}{2} + \dfrac{\dot{m}_{a_{fan}}(U_{1e} - U_0)^2}{2}\right] + \left[\dfrac{\dot{m}_f(U_e - U_0)^2}{2} - \dfrac{\dot{m}_f(U_0)^2}{2}\right]}{\dot{m}_f h_f} \tag{5.42}$$

If both exhausts are fully expanded so that $P_e = P_0 \;\; ; \;\; P_{1e} = P_0$ the thermal efficiency becomes

$$\eta_{th} = \frac{(\dot{m}_{a_{core}}(U_e - U_0) + \dot{m}_{a_{fan}}(U_{1e} - U_0) + \dot{m}_f U_e)U_0}{\dot{m}_f h_f} +$$
$$\frac{\left[\dfrac{\dot{m}_{a_{core}}(U_e - U_0)^2}{2} + \dfrac{\dot{m}_{a_{fan}}(U_{1e} - U_0)^2}{2}\right] + \left[\dfrac{\dot{m}_f(U_e - U_0)^2}{2} - \dfrac{\dot{m}_f(U_0)^2}{2}\right]}{\dot{m}_f h_f} \tag{5.43}$$

which reduces to

$$\eta_{th} = \frac{\dot{m}_{a_{core}}\left(\dfrac{U_e^{\,2}}{2} - \dfrac{U_0^2}{2}\right) + \dot{m}_f \dfrac{U_e^2}{2} + \dot{m}_{a_{fan}}\left(\dfrac{U_{1e}^{\,2}}{2} - \dfrac{U_0^2}{2}\right)}{\dot{m}_f h_f} \tag{5.44}$$

We can recast (5.44) in terms of enthalpies using the following relations

$$\dot{m}_f(h_f - h_{t4}) = \dot{m}_{a_{core}}(h_{t4} - h_{t3})$$

$$h_{t5} = h_e + \frac{U_e^2}{2} \tag{5.45}$$

$$h_{t13} = h_{1e} + \frac{U_{1e}^2}{2}$$

where the fan and core nozzle streams are assumed to be adiabatic. Now

$$\eta_{th} = \frac{[\dot{m}_{a_{core}}((h_{t5} - h_e) - (h_{t0} - h_0)) + \dot{m}_{a_{fan}}((h_{t13} - h_{1e}) - (h_{t0} - h_0)) + \dot{m}_f(h_{t5} - h_e)]}{(\dot{m}_f + \dot{m}_{a_{core}})h_{t4} - \dot{m}_{a_{core}}h_{t3}} \tag{5.46}$$

Rearrange (5.46) to read

$$\eta_{th} = \frac{(\dot{m}_{a_{core}} + \dot{m}_f)h_{t5} + \dot{m}_{a_{fan}}h_{t13} - (\dot{m}_{a_{core}} + \dot{m}_{a_{fan}})h_{t0}}{(\dot{m}_f + \dot{m}_{a_{core}})h_{t4} - \dot{m}_{a_{core}}h_{t3}} - \frac{\dot{m}_{a_{core}}(h_e - h_0) + \dot{m}_{a_{fan}}(h_{1e} - h_0) + \dot{m}_f h_e}{(\dot{m}_f + \dot{m}_{a_{core}})h_{t4} - \dot{m}_{a_{core}}h_{t3}} \qquad (5.47)$$

Recall the turbofan work balance (5.27). This relation can be rearranged to read

$$(\dot{m}_f + \dot{m}_{a_{core}})h_{t4} - \dot{m}_{a_{core}}h_{t3} = (\dot{m}_{a_{core}} + \dot{m}_f)h_{t5} + \dot{m}_{a_{fan}}h_{t13} - (\dot{m}_{a_{core}} + \dot{m}_{a_{fan}})h_{t0} \qquad (5.48)$$

where it has been assumed that the inlet is adiabatic $h_{t2} = h_{t0}$. Now use (5.48) to replace the numerator or denominator in the first term of (5.47). The thermal efficiency finally reads

$$\eta_{th} = 1 - \frac{Q_{\text{rejected during the cycle}}}{Q_{\text{input during the cycle}}} = 1 - \frac{(\dot{m}_{a_{core}} + \dot{m}_f)(h_e - h_0) + \dot{m}_{a_{fan}}(h_{1e} - h_0) + \dot{m}_f h_0}{(\dot{m}_f + \dot{m}_{a_{core}})h_{t4} - \dot{m}_{a_{core}}h_{t3}} \qquad (5.49)$$

The expression in (5.49) for the heat rejected during the cycle,

$$Q_{\text{rejected during the cycle}} = (\dot{m}_{a_{core}} + \dot{m}_f)(h_e - h_0) + \dot{m}_{a_{fan}}(h_{1e} - h_0) + \dot{m}_f h_0 \qquad (5.50)$$

brings to mind the discussion of thermal efficiency in Chapter 2. The heat rejected comprises heat conduction to the surrounding atmosphere from the fan and core mass flows plus physical removal from the thermally equilibrated nozzle flow of a portion equal to the added fuel mass flow. From this perspective the added fuel mass carries its fuel enthalpy into the system and the exhausted fuel mass carries its ambient enthalpy out of the system and there is no net mass increase or decrease to the system.

The main assumptions underlying (5.49) are that the engine operates adiabatically, the shaft mechanical efficiency is one and the burner combustion efficiency is one. Engine components are not assumed to operate ideally - they are not assumed to be isentropic.

5.4.1 THERMAL EFFICIENCY OF THE IDEAL TURBOFAN

For the ideal cycle assuming constant C_p equation (5.49) becomes, in terms of temperatures,

$$\eta_{th_{\text{ideal turbofan}}} = 1 - \frac{(1 + (1 + \beta)f)T_e - T_0}{(1 + (1 + \beta)f)T_{t4} - T_{t3}} = 1 - \left(\frac{1}{\tau_r \tau_c}\right)\left(\frac{(1 + (1 + \beta)f)\frac{T_e}{T_0} - 1}{(1 + (1 + \beta)f)\frac{\tau_\lambda}{\tau_r \tau_c} - 1}\right). \qquad (5.51)$$

Using (5.25) Equation (5.51) becomes

$$\eta_{th_{\text{ideal turbofan}}} = 1 - \left(\frac{1}{\tau_r \tau_c}\right) \qquad (5.52)$$

which is identical to the thermal efficiency of the ideal turbojet. Notice that for the ideal turbofan with $h_{1e} = h_0$ the heat rejected by the fan stream is zero. Therefore the thermal efficiency of the ideal turbofan is independent of the parameters of the fan stream.

5.5 THE NON-IDEAL TURBOFAN

The fan, compressor and turbine polytropic relations are

$$\pi_{c1} = \tau_{c1}^{\frac{\gamma\eta_{pc'}}{\gamma-1}} \qquad \pi_c = \tau_c^{\frac{\gamma\eta_{pc}}{\gamma-1}} \qquad \pi_t = \tau_t^{\frac{\gamma}{(\gamma-1)\eta_{pe}}} \tag{5.53}$$

where $\eta_{pc'}$ is the polytropic efficiency of the fan. The polytropic efficiencies η_{pc}, $\eta_{pc'}$ and η_{pe} are all less than one. The inlet, burner and nozzles all operate with some stagnation pressure loss.

$$\pi_d < 1 \qquad \pi_{n1} < 1 \qquad \pi_n < 1 \qquad \pi_b < 1\,. \tag{5.54}$$

5.5.1 NON-IDEAL FAN STREAM

The stagnation pressure ratio across the fan is

$$P_{t1e} = P_0\pi_r\pi_d\pi_{c1}\pi_{n1} = P_{1e}\left(1 + \frac{\gamma-1}{2}M_{1e}^2\right)^{\frac{\gamma}{\gamma-1}}. \tag{5.55}$$

The fan nozzle is still assumed to be fully expanded and so the Mach number ratio for the non-ideal turbofan is

$$\frac{M_{1e}^2}{M_0^2} = \frac{\tau_r\tau_{c1}^{\eta_{pc'}}(\pi_d\pi_{n1})^{\frac{\gamma-1}{\gamma}} - 1}{\tau_r - 1}. \tag{5.56}$$

The stagnation temperature is (assuming the inlet and fan nozzle are adiabatic)

$$T_{t1e} = T_0\tau_r\tau_{c1} = T_{1e}\left(1 + \frac{\gamma-1}{2}M_{1e}^2\right) = T_{1e}\tau_r\tau_{c1}^{\eta_{pc'}}(\pi_d\pi_{n1})^{\frac{\gamma-1}{\gamma}} \tag{5.57}$$

and

$$\frac{T_{1e}}{T_0} = \frac{\tau_{c1}^{1-\eta_{pc'}}}{(\pi_d\pi_{n1})^{\frac{\gamma-1}{\gamma}}}. \tag{5.58}$$

Now the velocity ratio across the non-ideal fan is

$$\frac{U_{1e}^2}{U_0^2} = \frac{1}{\tau_r - 1}\left(\tau_r \tau_{c1} - \frac{\tau_{c1}^{1-\eta_{pc'}}}{(\pi_d \pi_{n1})^{\frac{\gamma-1}{\gamma}}}\right). \tag{5.59}$$

5.5.2 Non-ideal core stream

The stagnation pressure across the core is.

$$P_{te} = P_0 \pi_r \pi_d \pi_c \pi_b \pi_t \pi_n = P_e\left(1 + \frac{\gamma-1}{2} M_e^2\right)^{\frac{\gamma}{\gamma-1}}. \tag{5.60}$$

The core nozzle is fully expanded and so the Mach number ratio is

$$\frac{M_e^2}{M_0^2} = \frac{\tau_r \tau_c^{\eta_{pc}} \tau_t^{\frac{1}{\eta_{pe}}} (\pi_d \pi_b \pi_n)^{\frac{\gamma-1}{\gamma}} - 1}{\tau_r - 1}. \tag{5.61}$$

In the non-ideal turbofan we continue to assume that the diffuser and nozzle flows are adiabatic and so

$$T_{te} = T_0 \tau_r \tau_c \tau_b \tau_t = T_e \tau_r \tau_c^{\eta_{pc}} \tau_t^{\frac{1}{\eta_{pe}}} (\pi_d \pi_b \pi_n)^{\frac{\gamma-1}{\gamma}} \tag{5.62}$$

from which is determined

$$\frac{T_e}{T_0} = \frac{\tau_c^{1-\eta_{pc}} \tau_t^{\left(1 - \frac{1}{\eta_{pe}}\right)} \tau_\lambda}{\tau_r \tau_c (\pi_d \pi_b \pi_n)^{\frac{\gamma-1}{\gamma}}}. \tag{5.63}$$

The velocity ratio across the core is

$$\left(\frac{U_e}{U_0}\right)^2 = \frac{1}{\tau_r - 1}\left(\tau_\lambda \tau_t - \frac{\tau_c^{1-\eta_{pc}} \tau_t^{\left(1 - \frac{1}{\eta_{pe}}\right)} \tau_\lambda}{\tau_r \tau_c (\pi_d \pi_b \pi_n)^{\frac{\gamma-1}{\gamma}}}\right). \tag{5.64}$$

The work balance across the engine remains essentially the same as in the ideal cycle

$$\tau_t = 1 - \frac{\tau_r}{\eta_m \tau_\lambda}\{(\tau_c - 1) + \beta(\tau_{c1} - 1)\} \tag{5.65}$$

where a shaft mechanical efficiency has been introduced defined as

$$\eta_m = \frac{\dot{m}_{a_{core}}(h_{t3} - h_{t2}) + \dot{m}_{a_{fan}}(h_{t13} - h_{t2})}{(\dot{m}_{a_{core}} + \dot{m}_f)(h_{t4} - h_{t5})}. \tag{5.66}$$

5.5.3 Maximum specific impulse non-ideal cycle

Equation (5.35) remains the same as for the ideal cycle.

$$\frac{1}{2(U_e/U_0)}\frac{\partial}{\partial\beta}\left(\frac{U_e^2}{U_0^2}\right) + \left(\frac{U_{e1}}{U_0} - 1\right) = 0. \tag{5.67}$$

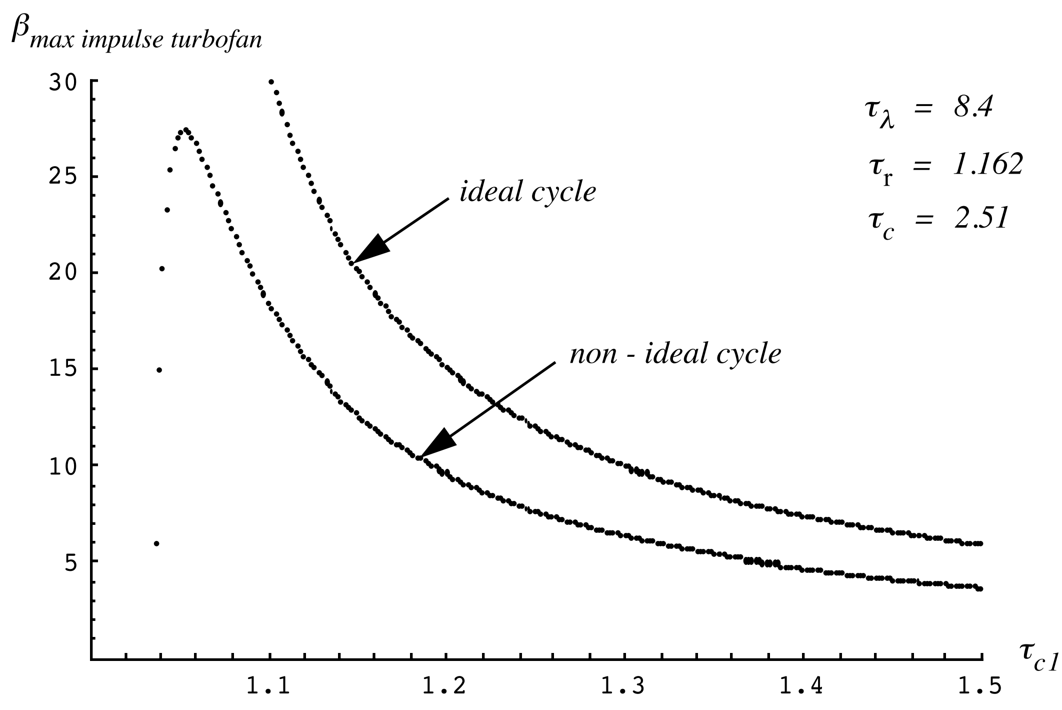


Figure 5.4 Turbofan bypass ratio for maximum specific impulse as a function of fan temperature ratio comparing the ideal with a non-ideal cycle. parameters of the nonideal cycle are $\pi_d = 0.95$, $\eta_{pc1} = 0.86$, $\pi_{n1} = 0.96$, $\eta_{pc} = 0.86$, $\pi_b = 0.95$, $\eta_m = 0.98$, $\eta_{pe} = 0.86$, $\pi_n = 0.96$.

The derivative is

$$\frac{\partial}{\partial \beta}\left(\frac{U_e^2}{U_0^2}\right) = \frac{-\tau_r(\tau_{c1}-1)}{\eta_m(\tau_r-1)}\left(1-\frac{\left(1-\frac{1}{\eta_{pe}}\right)\tau_c^{\,1-\eta_{pc}}\,\tau_t^{-\frac{1}{\eta_{pe}}}}{\tau_r\tau_c(\pi_d\pi_b\pi_n)^{\frac{\gamma-1}{\gamma}}}\right). \tag{5.68}$$

Equations (5.59), (5.64), (5.65) and (5.68) are inserted into (5.67) and the optimal bypass ratio for a set of selected engine parameters is determined implicitly. A typical numerically determined result is shown in Figure 5.4 and Figure 5.5.

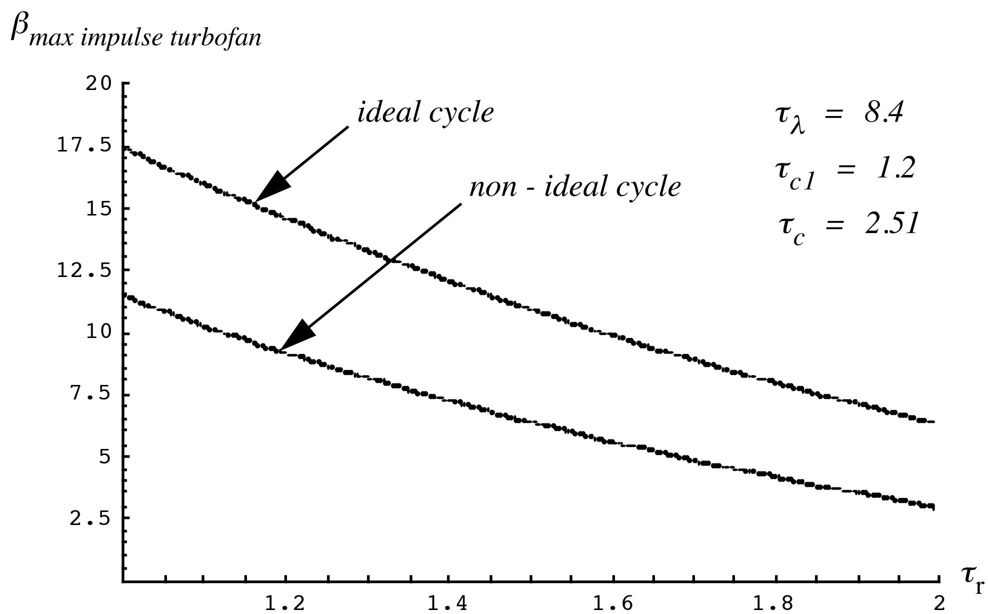


Figure 5.5 Turbofan bypass ratio for maximum specific impulse as a function of Mach number comparing the ideal with a non-ideal cycle. Parameters of the nonideal cycle are $\pi_d = 0.95$, $\eta_{pc1} = 0.86$, $\pi_{n1} = 0.96$, $\eta_{pc} = 0.86$, $\pi_b = 0.95$, $\eta_m = 0.98$, $\eta_{pe} = 0.86$, $\pi_n = 0.96$.

These figures illustrate the strong dependence of the optimum bypass ratio on the non-ideal behavior of the engine. In general as the losses increase, the bypass ratio optimizes at a lower value. But note that the optimum bypass ratio of the non-ideal engine is still somewhat higher than the values generally used in real engines. The reason for this is that our analysis does not include the optimization issues connected to integrating the engine onto an aircraft where there is a premium on designing to a low frontal area so as to reduce drag while maintaining a certain clearance between the engine and the runway.

Nevertheless our analysis helps us to understand the historical trend toward higher bypass engines as turbine and fan efficiencies have improved along with increases in the turbine inlet temperature.

5.6 PROBLEMS

Problem 1 - Assume $\gamma = 1.4$, $R = 287$ M^2/(sec^2-°K), $C_p = 1005$ M^2/(sec^2-°K). The fuel heating value is 4.28×10^7 J/Kg. Where appropriate assume $f \ll 1$. The ambient temperature and pressure are $T_0 = 216K$ and $P_0 = 2 \times 10^4 N/M^2$. Consider a turbofan with the following characteristics.

$$M_0 = 0.85 \;; \quad \tau_\lambda = 8.0 \;; \quad \pi_c = 30 \;; \quad \pi_{c1} = 1.6 \;; \quad \beta = 5 . \tag{5.69}$$

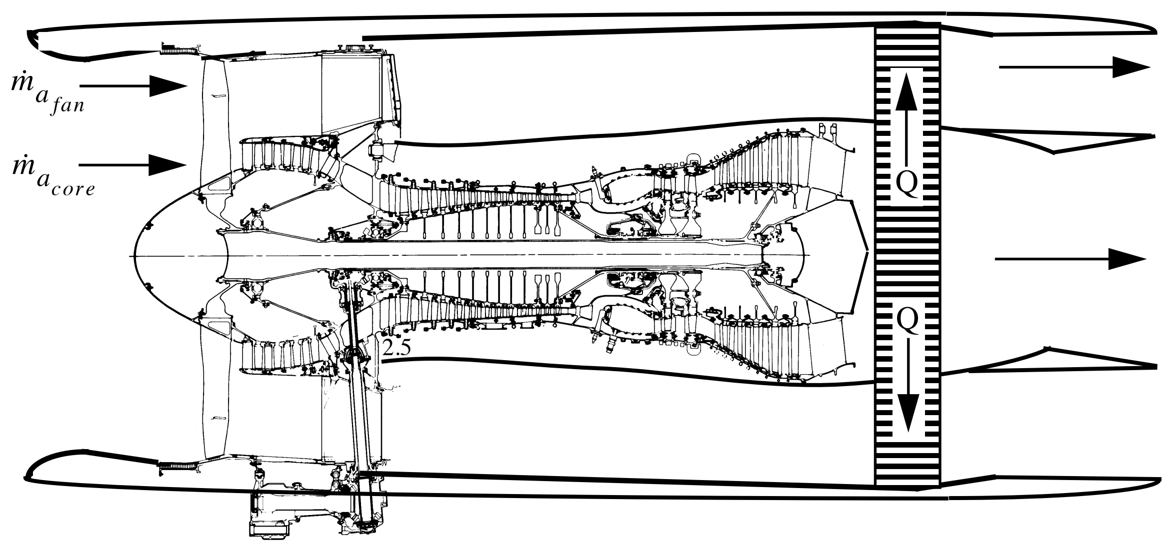
The compressor, fan and turbine polytropic efficiencies are

$$\eta_{pc} = 0.9 \;; \quad \eta_{pc1} = 0.9 \;; \quad \eta_{pt} = 0.95 . \tag{5.70}$$

Let the burner efficiency and pressure ratio be $\eta_b = 0.99 \;; \quad \pi_b = 0.97$. Assume the shaft efficiency is one. Both the fan and core streams use ideal simple convergent nozzles. Determine the dimensionless thrust $T/(P_0 A_0)$, specific fuel consumption and overall efficiency. Suppose the engine is expected to deliver 8,000 pounds of thrust at cruise conditions. What must be the area of the fan face, A_2?

Problem 2 - Use Matlab or Mathematica to develop a program that reproduces Figure 5.4 and Figure 5.5.

Problem 3 - An ideal turbofan operates with a heat exchanger at its aft end.



The heat exchanger causes a certain amount of thermal energy Q (Joules/sec) to be transferred from the hot core stream to the cooler fan stream. Let the subscript x refer to the heat exchanger.

Assume that the heat exchanger operates without any loss of stagnation pressure $\pi_x = \pi_x' = 1.0$ and that both nozzles are fully expanded. Let $\tau_x = T_{te}/T_{t5}$ and $\tau_x' = T_{te}'/T_{t5}'$. The thrust is given by

$$T = \dot{m}_{a_{core}} a_0 M_0 \{(U_6/U_0 - 1) + \beta(U_6'/U_0 - 1)\} \tag{5.71}$$

where we have assumed $f \ll 1$.

1) Derive an expression for $T/(\dot{m}_{a_{core}} a_0)$ in terms of τ_λ, τ_r, τ_c, τ_c', β and τ_x, τ_x'.

2) Write down an energy balance between the core and fan streams. Suppose an amount of heat Q is exchanged. Let $\tau_x = 1 - \alpha$ where $\alpha = Q/(\dot{m}_{a_{core}} C_p T_{t5})$, $Q > 0$. Show that

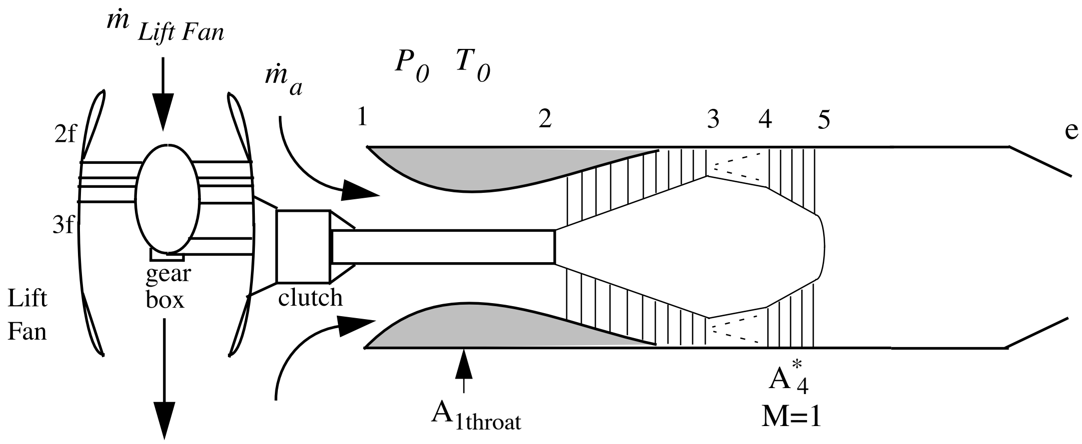
$$\tau_x' = 1 + \left(\frac{\tau_\lambda \tau_t}{\beta \tau_r \tau_c'}\right)\alpha . \tag{5.72}$$

3) Consider an ideal turbofan with the following characteristics.

$$T_0 = 216K \ ; \ M_0 = 0.85 \ ; \ \tau_\lambda = 7.5 \ ; \ \pi_c = 30 \ ; \ \pi_c' = 1.6 \ ; \ \beta = 5 . \tag{5.73}$$

Plot $T/(\dot{m}_{a_{core}} a_0)$ versus α for $0 < \alpha < \alpha_{max}$ where α_{max} corresponds to the value of α such that the two streams are brought to the same stagnation temperature coming out of the heat exchanger.

Problem 4 - The figure below shows a turbojet engine supplying shaft power to a lift fan. Assume that there are no mechanical losses in the shaft but the clutch and gear box that transfers power to the fan has an efficiency of 80%. That is, only 80% of the shaft power is used to increase the enthalpy of the air flow through the lift fan. The air mass flow rate through the lift fan is equal to twice the air mass flow rate through the engine $\dot{m}_{Lift\ Fan} = 2\dot{m}_a$. The polytropic efficiency of the lift fan is $\eta_{p\ Lift\ Fan} = 0.9$ and the air flow through the lift fan is all subsonic. The flight speed is zero.



The ambient temperature and pressure are $T_0 = 300K$ and $P_0 = 1.01x10^5 N/M^2$. The turbine inlet temperature is $T_{t4} = 1800K$ and $\pi_c = 25$. Relevant area ratios are $A_2/A_4^* = 15$ and $A_{1throat}/A_2 = 0.5$. Assume the compressor, burner and turbine all operate ideally. The nozzle is a simple convergent design and stagnation pressure losses due to wall friction in the inlet and nozzle are negligible. Assume f<<1. Let the nozzle area be set so that $P_{t5}/P_0 = 3$.

1) Is there a shock in the inlet of the turbojet?

2) Determine the stagnation temperature and pressure ratio across the lift fan

$$\tau_{Lift\ Fan} = \frac{T_{t3\ LiftFan}}{T_{t2\ LiftFan}} \qquad \pi_{Lift\ Fan} = \frac{P_{t3\ LiftFan}}{P_{t2\ LiftFan}} \tag{74}$$